Homophily, Selection, and Choice in Segregation Models

Bing Xu[1], William A.V. Clark[2], Maxwell Pak[3]

December 2023

Abstract

Schelling's 1971 work on the dynamics of segregation showed that even a small degree of homophily, the desire to live among like neighbors, can lead to a starkly segregated population. One of the driving factors for this result is that the notion of homophily used is based on group identities that are exogenous and immutable. In contrast, we consider a homophily that arises from the desire to be with neighbors who are behaviorally similar, not necessarily those who have the same group identity. The distinction matters because behaviors are neither exogenous nor immutable but choices that can change as individuals adapt to their neighborhoods. We show that in such an environment integration rather than segregation is the typical outcome. However, the tendency toward adaptation and integration can be impeded when economic frictions in the form of income inequality and housing cost are present.

Significance: Understanding the drivers of residential segregation is important for its mitigation. Schelling's influential work in 1971 investigating the role of homophily in residential dynamics is often used as a framework for investigating residential segregation. However, because Schelling's model results in segregation so robustly, it is difficult to understand the factors that may mitigate or exacerbate segregation. Our work shows that if homophily is based not on individuals' attributes that are immutable and instead on behaviors, and individuals are given a chance to adapt their behaviors to their neighborhood, then the adaptation process yields integration as the typical outcome. Furthermore, the tendency toward integration may be impeded by the presence of housing cost and income disparity between the majority and the minority groups.

---

[1] Southwestern University of Finance and Economics, China.
[2] University of California, Los Angeles, United States.
[3] Southwestern University of Finance and Economics, China.

1 Introduction

Schelling's model of segregation (1971) showed that individuals making residential choices based on homophily, the desire to be with those who are similar to ourselves, can lead to a starkly segregated population.[4] That conclusion became important in the serious debate that started in the 1980s about whether it is housing discrimination or individual preference that is primarily responsible for the continuing racial residential segregation in the U.S. In the past half century since the formulation of Schelling's model, however, attitudes and behavior toward race have changed, and segregation patterns have become more complex in reflection of those changes, improving in many places while worsening in others (figure 1). Accordingly, there have been many investigations into residential patterns generated by Schelling's model under various sets of individual preferences, including wide-ranging degrees of homophily and affinity for wealth and status, using both mathematically tractable models (Zhang 2004; Zhang 2011) and simulations of agent-based models (Berg et al. 2010; Bernard and Willer 2007; Clark and Fossett 2008; Fossett 2011; Li et al. 2020; Pancs and Vriend 2007). Some of these works, such as Clark and Fossett (2008) and Fossett (2011), have tried to replicate the complex and subtle residential patterns observed in real urban environments, while others have studied how even a very mild degree of homophily can lead to a highly segregated population (Li et al. 2020; Pancs and Vriend 2007; Zhang 2004; Zhang 2011).[5] Pancs and Vriend (2007) in particular have shown that segregation occurs robustly even when individuals strictly prefer to live in a fully integrated neighborhood (each group comprising 50% of the neighborhood) if they prefer to be in the numerical majority rather than in the minority when such a neighborhood is not available.[6] In contrast to the study on the consequences of homophily, there have been relatively few investigations into its origin, a notable exception being Jasso (2010) who show how sociobehavioral processes of justice, status, and power can induce a wide-ranging preference for integration and segregation.

---

[4] Although Schelling's model is a socio-economic model that can be interpreted naturally under Jasso (2000)'s broad framework linking individual micro behaviors and macro social phenomena, Vinkovic and Kirman (2006) have shown that it can also be viewed from a physics perspective via a clustering model.

[5] Berg et al. (2010) provides a modification of Schelling's model that induces less segregation. However, that modification has individuals basing their relocation decision on the prior experience of having lived together with their potential new neighbors. Such an approach is somewhat limiting in that it requires that there are sufficiently many neighbors in the new neighborhood under consideration that the individual recognizes as friends, or non-friends, from her past, which seems unlikely in an actual urban environment.

[6] As Pancs and Vriend note, fully integrated neighborhoods occur rarely, so the individuals end up mostly choosing between neighborhoods where they will be in the majority or in the minority. Thus, as long as individuals prefer to be in the majority rather than in the minority, Schelling-like dynamic results, and the population end up segregated.

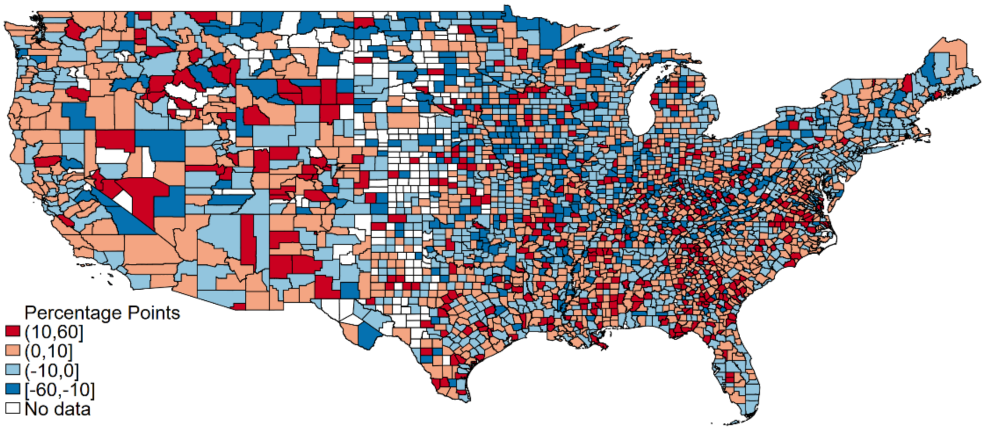


Figure 1: Changes in the segregation level (dissimilarity index), 2009-2020.

This paper follows the tradition of studying residential patterns under Schelling's framework but considers an aspect of homophily that has received limited attention. Schelling's model was developed at a time of strong ethnic divisions and the continuing effects of racial residential segregation in U.S. cities. Consequently, his model and the models in this vein feature agents who differ on some measure of identity: black or white, rich versus poor, English or French speaking, or high versus low status. Because the identities are assumed to be exogenous and immutable in these models, acculturalization, which is one of the key drivers of spatial assimilation (Charles 2003; Crowell and Fossett 2022), falls outside the scope of the models. In contrast, this paper considers a homophily that is based on behavioral norms. The distinction matters because behaviors are neither exogenous nor immutable but choices individuals make and can change as they adapt to their environment. Fu et al. (2012) note that homophily may have emerged from an evolutionary process because it confers an advantage in certain contexts. In a similar spirit, our model assumes that there are two ways to behave in some dimension and that individuals enjoy a higher utility when they interact with neighbors who behave similarly as they do. Some examples include, language spoken, religion practiced, emphasis placed on children's education, and effort made in maintaining the appearance of their houses.[7] We further posit that different groups have a different modal behavior so that wanting to be with neighbors of the same group can be interpreted as a proxy for wanting to be with neighbors who are behaviorally similar.

A coordination game provides a natural vehicle to model such neighborhood interactions.[8] By having the individuals repeatedly interact with their neighbors in the coordination game

---

[7] As these examples make clear, some behavioral norms are more adaptable than others. Our model focuses on behaviors that are relatively easy to change, or at least ones, like language, that an individual is willing to change even at a greater cost. There may also be situations where initial bias is so strong that individuals may be reluctant to engage with members of the other group even after behavioral change. A richer model may explore how such frictions in adaptation affects segregation dynamics.

[8] A coordination game models a situation where a player earns a higher payoff if she selects the same course of action as the other players. Because coordination games have multiple (pure strategy) Nash equilibria, all of

as they make their residential choices and allowing them to revise their behavioral choice based on their experience, we examine whether individuals of different groups will "learn to live together" by adopting a common behavioral norm or whether they will maintain their initial predispositions and remain segregated. Our simulation results show that if there is some randomness in the population so that individuals sometimes move for idiosyncratic reasons other than homophily and some inertia so that individuals do not move again immediately after moving into a neighborhood, then integration rather than segregation is the typical outcome, just as we would expect from the changes in the attitudinal responses to race and ethnicity.[9]

To study how this outcome is altered in the presence of economic forces, we incorporate income inequality and housing cost into the base model described above. More specifically, individuals have heterogeneous income, with the numerical majority group having a higher average income, and derive utility from consumption and housing.[10] The utility from housing is obtained from the neighborhood interaction specified in the base model while consumption is income minus the cost of renting a house. Residential locations are divided into two districts of equal size, and individuals make their relocation decisions by comparing the utility they expect to obtain in each district.[11] The rental price, which is assumed to be uniform within a district, is in turn determined by the demand for housing in the district. When income and housing cost are introduced in this manner, the resulting simulations typically show a higher level of segregation.

Although others, such as Bernard and Willer (2007), Malmberg and Clark (2020), and Sethi and Somanathan (2004), have also investigated the effects of wealth and status on residential patterns, the dynamics in our model are different. In the earlier works, living among the wealthy and high-status individuals is assumed to be inherently desirable because, for example, it offers better schools and amenities or confers a higher status on the individual. As a result, there is a one-sided homophily, in which wealthy and high-status individuals prefer and choose to live together. Their choices induce higher housing prices in their district, which their higher income allows them to afford but prevent poor and low-status individuals from joining them. Thus, the combined effects of choice and exclusion amplify the race or ethnicity-based segregation when wealth and status are strongly correlated with race or ethnicity but attenuate it when the correlation is weak. In contrast, we deliberately exclude

---

which involve players acting identically, they serve as a useful framework for studying how social norms arise (Conley and Neilson, 2013).

[9] Although integration in the context of residential segregation is a positive outcome, it is important to note that the desirability of integration is context dependent. For example, assimilation may be viewed negatively as a loss of a culture. The results presented in this paper are agnostic on whether integration is positive or negative.

[10] We focus on the case in which the numerical majority is the advantaged group. Extending our study to a more general case where either the majority or the minority may be the advantaged group, as in Jasso (2010, 2023), may yield important additional insights, and this point discussed further in the supplement to our paper.

[11] The term "neighborhood" is used in this paper to denote the geographic scope of an individual's interaction while "district" denotes a set of residential locations separated by a static boundary.

affinity for living in a wealthy neighborhood to keep it from confounding the purely economic effects of wealth and housing cost.[12] Nevertheless, income disparity still induces segregation through housing price in our model. Segregation then entrenches the behavioral norms of the two groups, which further creates the incentive to segregate and maintains the price difference.

At a broader level, our paper also relates to the rich literature on economic evolution and adaptation, in which an agent's characteristics or choices depend on those of other agents. For example, Föllmer (1974)'s model of pure exchange economy features individuals whose preference depends on their neighbors' preferences. Like our work, Föllmer studies the extent to which neighborhood interactions affect the macro level outcome, the level of segregation in our case and the existence of a Walrasian equilibrium in Föllmer's. The literature on herding in financial markets is particularly close to our work in that both model agents whose action choices are directly affected by the choices, not just the characteristics, of others. For example, in the more recent work of Föllmer et al. (2005), investors choose one of several price-forecasting rules to trade a risky asset, and the authors study the macro level outcome, the price of the asset, when the investors revise their choice based on its performance, which like our model depends on the choices of others.[13]

2 Base model

The population consists of two groups of individuals, group A (majority) and group B (minority). Each individual occupies a location (square) on a $N \times N$ grid, and each location contains at most one individual. A neighborhood consists of a location and its adjacent squares.[14] Time is discrete, and in each period individuals interact with each one of their neighbors by playing a coordination game. In particular, individuals simultaneously and independently choose one of two actions, a or b, that they will play during that period. The groups are distinguished by their initial propensities: group A individuals have a predisposition toward playing a while group B individuals have a predisposition toward b. If an individual and her

---

[12] It is possible to incorporate the desirability of high-income neighborhoods by making an individual's utility an increasing function of the average neighborhood income. Given the assumed income disparity between the majority and the minority, such a modification should make the segregation tendency seen in our income and housing cost model stronger and not change our results qualitatively.

[13] At each period, the investors in Föllmer et al. (2005) form demand for the asset based on their forecasted prices, and the realized price is the market-clearing price. They evaluate their choice's performance, which depends on the forecast model chosen by the others, and stochastically revise their choice. The authors showed that the resulting price process is ergodic under a reasonable condition and generates salient features of the financial-market time series, such as recurrent "bubbles" and the subsequent returns to the fundamentals, without investors being systemically wrong.

[14] In keeping with Schelling's original work, we assume that the grid is a simple square, rather than a torus, where the top and the bottom edges and the left and the right edges are joined. Therefore, locations in the interior of the grid have eight neighboring squares while those on the edges of the grid have less.

neighbor both chose a or both chose b, they both get a coordination utility. If they miscoordinate, that is, one plays a while the other plays b, they get a lower, miscoordination utility.

To keep the two groups as symmetric as possible, we assume that there is no inherent advantage to coordinating on a versus b for either group and similarly for miscoordinating.[15] More precisely, letting $\pi_i(a_i, a_j)$ denote the utility individual i receives whens she plays action $a_i \in \{a, b\}$ while her neighbor j plays $a_j \in \{\mathrm{a}, \mathrm{b}\}$, we set $\pi_i(a, a) = \pi_i(b, b) = \pi_c$ and $\pi_i(a, b) = \pi_i(b, a) = \pi_m$, where $\pi_c > \pi_m > 0$. The utilities the individuals receive from the interaction is summarized in the payoff matrix below, where rows represent the individual's choice, and the columns represent the neighbor's choice. The first number in the boxes represents the utility the individual receives if the corresponding row and column are chosen, and the second number represents the neighbor's utility.

		neighbor	
		a	b
individual	a	π_c, π_c	π_{m}, π_m
	b	π_m, π_m	π_c, π_c

2.1 Model dynamics

The model dynamics are described below. To ease the exposition, the rules that govern an individual's within-period behavior is specified separately from the ones that govern her relocation decisions.

Within-period behavior: In each period $t \geq 1$, individuals have current assessments for the two actions, $\tilde{\pi}_{it}(a)$ and $\tilde{\pi}_{it}(b)$, which represent the utilities they expect to receive from taking the actions and are based on their initial propensities and the average utilities they received when they had chosen the actions in the past. Individuals choose an action to play in the current period according to a logit model. That is, letting a_{it} be the action chosen by individual i at period t,

$$\mathrm{Prob}\,(a_{it} = a) = \frac{e^{\tilde{\pi}_{it}(a)}}{e^{\tilde{\pi}_{it}(a)} + e^{\tilde{\pi}_{it}(b)}} \quad \text{and} \quad \mathrm{Prob}\,(a_{it} = b) = \frac{e^{\tilde{\pi}_{it}(b)}}{e^{\tilde{\pi}_{it}(a)} + e^{\tilde{\pi}_{it}(b)}}.$$

Individuals then play the coordination game against each of their neighbors, using the same action that was chosen in the beginning of the period. An individual's period-t utility is defined as the average utility she has received in the coordination game with her neighbors:

---

[15] In the supplement to this paper, we consider a number of variations of our model as a robustness check. In one variation, we assume that individuals have a persistent preference for their own group's action and show that the resulting segregation pattern is qualitatively similar, except in cases where the preference for their own group's action is so strong that minority individuals would not want to switch to the majority action in a one-shot version of the coordination game.

$$\pi_{it} = \frac{\sum_{j \in \mathcal{N}_{it}} \pi_i \left(a_{it}, a_{jt}\right)}{|\mathcal{N}_{it}|},$$

where $\mathcal{N}_{it}$ is her current neighbors and $|\mathcal{N}_{it}|$ is the number of the neighbors. If an individual has no neighbors, then π_{it} is set as $\tilde{\pi}_{it}(a_{it})$ so that her assessment for the chosen action does not change. At the end of the period, individuals' assessments for the actions they used are updated according to their own payoff experience. For example, if $a_{it} = a$, then

$$\tilde{\pi}_{it+1}(a) = \frac{N_{it}(a)\tilde{\pi}_{it}(a) + \pi_{it}}{N_{it}(a) + 1} \quad \text{and} \quad \tilde{\pi}_{it+1}(b) = \tilde{\pi}_{it}(b),$$

where $N_{it}(a)$ is the number of times individual i had chosen action a prior to period t.

To complete the behavioral rule, initial assessments, $\tilde{\pi}_{i1}(a)$ and $\tilde{\pi}_{i1}(b)$, and the number of times the actions are assumed to have been taken when the dynamic begins, $N_{i1}(a)$ and $N_{i1}(b)$, need to be specified. For $t \geq 2$, $\tilde{\pi}_{it}(a)$ can be expressed as

$$\tilde{\pi}_{it}(a) = \frac{N_{i1}(a)\tilde{\pi}_{i1}(a) + \sum_{k=1}^{t-1} \pi_{ik} \, \mathbf{1}_{(a_{ik}=a)}}{N_{i1}(a) + \sum_{k=1}^{t-1} \mathbf{1}_{(a_{ik}=a)}},$$

where $\mathbf{1}_{(a_{ik}=a)}$ is an indicator variable that takes value one if the action chosen in period k is a and value zero otherwise (and similarly for $\tilde{\pi}_{it}(b)$). Thus, the initial values determine the extent to which the initial predispositions toward the actions persist. The actual values for the simulations are chosen to reflect the individuals' group identities and are given in Section 2.2.

Relocation behavior: At the end of the period, the individuals make their relocation decision sequentially in random order. As in Schelling's model, they move when their happiness level falls below a threshold level. Unlike that model, however, individuals' happiness level is not determined by the number of neighbors who have the same group identity as themselves but by the number of neighbors who behave like themselves. In particular, individuals keep track of the utilities they have received since moving into a neighborhood, and if that average falls below a threshold level, $\bar{u}$, they are deemed unhappy. If, in addition, they have lived in the neighborhood for at least a preset number of periods, $\bar{t}$, then they move to a new location. That is, if an individual moves to a neighborhood at period t_0, she will move at the end of period t if

$$\frac{\sum_{k=t_0}^{t} \pi_{ik}}{t - t_0 + 1} < \bar{u} \quad \text{and} \quad t - t_0 + 1 \geq \bar{t}.$$

The minimum tenure requirement is assumed in order to impart realism to the model and also to give individuals a chance to adapt to their neighborhoods.

We divide the grid into two equal-sized districts, left and right, and assume that when unhappy individuals relocate, they move to the district with a higher density of similarly behaving individuals. To be more precise, let $\mu_t(\tau)$ be the fraction of individuals who played action a in district τ in period t, and, with a slight abuse of notation, let

$$\tilde{\pi}_{it+1}(\tau) = \sum_{a_i \in \{a,b\}} \text{Prob}\,(a_{it+1} = a_i)\left(\pi_i(a_i, a)\mu_t(\tau) + \pi_i(a_i, b)\big(1 - \mu_t(\tau)\big)\right)$$

be the utility individual i expects to obtain by moving to district τ in the next period.[16] When an unhappy individual relocates, she moves to a random location in the district with the higher expected utility, provided that a vacant location exists.[17] If there is no vacancy, then she moves to a random location in the other district. Finally, individuals in our model also move for reasons other than homophily. In particular, individuals who have met the minimum tenure requirement have a small probability $p \geq 0$ of moving to a new randomly chosen location regardless of their happiness level.

2.2 Simulation and results

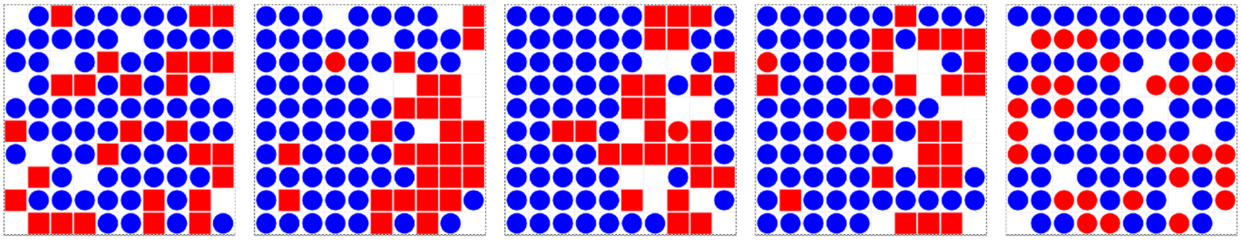
The simulation of our model is programmed in Python using Mesa agent-based modeling framework (https://github.com/projectmesa/mesa/). The coordination and the miscoordination payoffs are set as $\pi_c = 10$ and $\pi_m = 1$, respectively. The initial assessments are $\tilde{\pi}_{i1}(a) = 10$ and $\tilde{\pi}_{i1}(b) = 1$ if i is a member of group A and $\tilde{\pi}_{i1}(a) = 1$ and $\tilde{\pi}_{i1}(b) = 10$ otherwise. These values mean that group A individual has 0.9999 probability of playing a initially while group B individual has 0.9999 probability of playing b. Thus, we refer to a as the majority action and b as the minority action. The initial number of actions are set as $N_{i1}(a) = N_{i1}(b) = 10$. The remaining model parameters are population density, d, minority fraction, m, happiness threshold, $\bar{u}$, tenure threshold, $\bar{t}$, and (expected) random mover fraction, p.

The results from a single simulation run of the base model on a 10×10 grid is given in figure 2(a). The parameter values are $d = 0.9$, $m = 0.3$, $\bar{u} = 8$, $\bar{t} = 5$, and $p = 0.1$. In the figure, individuals occupy squares in the grid. Blue indicates group A while red indicates group B. A circle means the individual has chosen action a to use in the current period while square means action b. As the figure shows, the model does not generate the stark segregation seen in the Schelling model despite assuming a rather low level of tolerance. The two groups initially segregate as individuals relocate to districts with more behaviorally similar individuals. However, because there are some spillovers of the majority group into the minority concentrated district, as well as individuals who move for non-homophilic reasons, some interaction between the members of the two groups continue and end up inducing a high degree of adaptation and integration in the medium run. The figure shows group B individuals starting to adopt the majority action at period 100. By period 1000, all of the population have adopted the majority action, and the grid appears integrated.

---

[16] This assumes that individuals are boundedly rational in that they take the current composition of actions in a district as the composition they will encounter in that district next period, rather than trying to forecast how the composition may be updated by relocation.

[17] When an individual is indifferent between the two districts, one is chosen with equal probability.

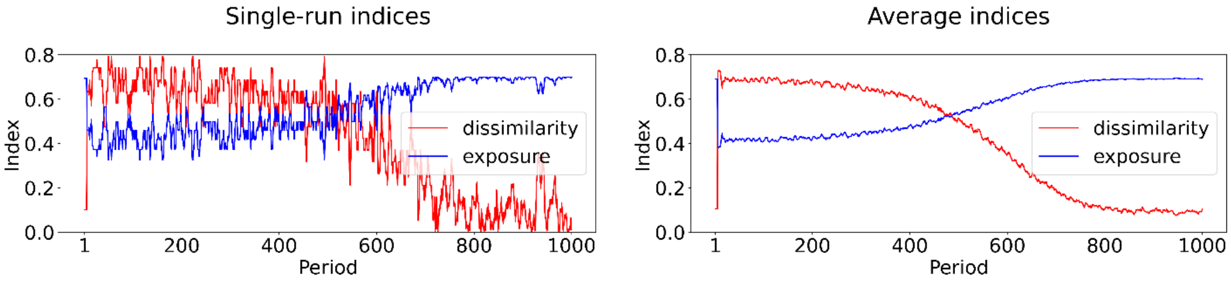


$t = 1$ $t = 10$ $t = 100$ $t = 500$ $t = 1000$

(a) Grid configuration: single run.

	t = 1	10	100	500	1000
dissimilarity: single	0.101	0.619	0.635	0.529	0.011
dissimilarity: average	0.104	0.723	0.694	0.513	0.103
exposure: single	0.694	0.473	0.460	0.533	0.700
exposure: average	0.691	0.387	0.412	0.531	0.690

(b) Dissimilarity and exposure indices.



(c) Dissimilarity and exposure indices.

Figure 2: Base model

To quantify the degree of segregation, we use the dissimilarity index, which measures the unevenness in the distribution of the two groups across districts, and the exposure index, which measures the degree of potential contact between the two groups within the districts. Recalling that the grid is divided into two districts, left (L) and right (R), the two indices are:

$$\begin{aligned} \text{dissimilarity index} &= \frac{1}{2} \sum_{\tau \in L,R} \left| \frac{A_\tau}{A_{\text{tot}}} - \frac{B_\tau}{B_{\text{tot}}} \right| \\ \text{exposure index} &= \sum_{\tau \in L,R} \left(\frac{B_\tau}{B_{\text{tot}}} \right) \left(\frac{A_\tau}{A_\tau + B_\tau} \right), \end{aligned}$$

where A_τ and A_{tot} are the number of group A individuals in district τ and in total, respectively, and B_τ and B_{tot} are analogous for group B individuals.[18] The indices corresponding to the simulation in figure 2(a), as well as the average indices for 100 simulations at the same parameter values, are given in figure 2(b), and the charts showing the indices at each period of the simulations are given in figure 2(c).

To explore how the model's parameters affect the residential pattern, we vary one parameter at a time while holding the remaining parameters at the values chosen above. One hundred simulations were run for each set of parameters. Figure 3 presents the two segregation indices at period 1000, averaged over the 100 simulations. In the range considered, higher population density seems to induce greater integration (panel (a)), while the fraction of the minority group in the population mostly has a negative effect on integration, with a large jump occurring at 30% (panel (b)). A higher happiness threshold, meaning less tolerance for behavioral difference, generally induces a greater segregation, but the effect appears quite weak at the chosen parameter values (panel (c)). Panels (d) and (e) show that some, but not much, tenure threshold and random movement are needed for integration to occur.

2.3 Discussion

Given an individual, call a neighborhood congruous if it has enough behaviorally similar neighbors to keep the individual happy. Each period spent in a congruous neighborhood reinforces the individual's current predispositions because interactions with her neighbors result in the coordination payoff sufficiently often. In contrast, residing in a non-congruous neighborhood has two opposing effects. Because the individual more often receives the miscoordination payoff, the individual's average per-period payoff may get pushed below the happiness threshold level and induce her to move in search of a more congruous neighborhood. At the same time, however, each encounter with a behaviorally dissimilar neighbor lowers the valuation of her current action choice relative to the other action. Thus, if the individual stays in a non-congruous neighborhood long enough, she may end up adopting the behavioral norm of her neighbors. Of course, if she moves to a more congruous neighborhood before the adaptation occurs, then the encounters with her new, behaviorally similar neighbors will once again reinforce her current predispositions. Thus, the degree to which the population segregates depends on the relative strengths of these forces.

---

[18] The exposure index is not symmetric, and, following the convention that views segregation as a separation from the dominant group (Crowell and Fossett 2022), we use the formulation measuring the degree to which a group B individual may encounter a group A individual in her district.

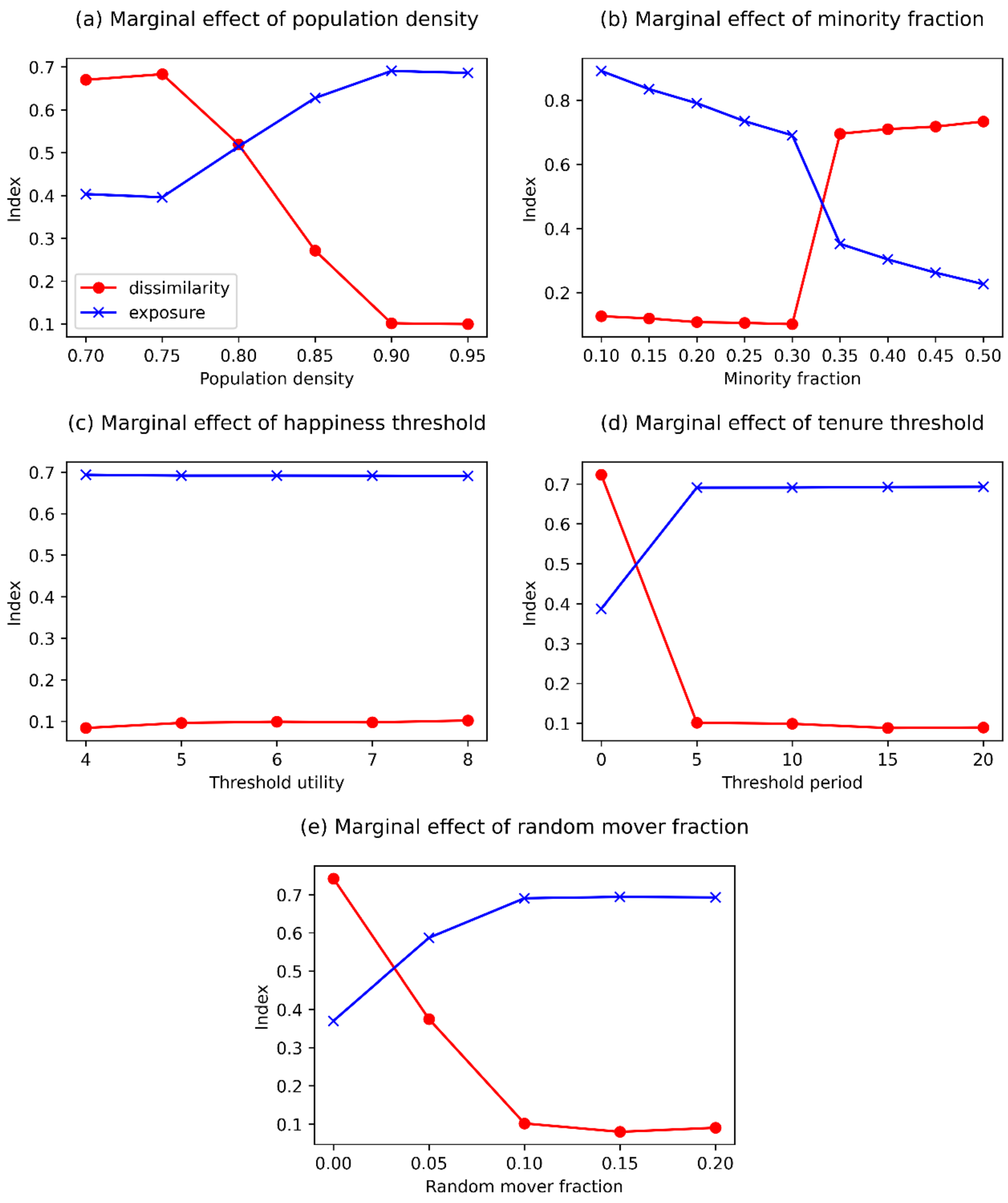


Figure 3: Marginal effects at chosen parameters, base model.

An explanation for why the adaptive force dominates and the minority group is assimilated in most of the simulations in figure 3 is the following. Because the minority group is outnumbered by the majority, minority members encounter more behaviorally dissimilar neighbors than similar neighbors on average, while the opposite is true for the majority group. This coupled with the tenure requirement, which forces a minimum number of interactions even with dissimilar neighbors, means that adaptation eventually dominates for the minority while reinforcement dominates for the majority. Thus, the entire population eventually adopts the majority favored action. Once such assimilation happens, relocation occurs only for idiosyncratic reasons and not for homophily, resulting in the population eventually becoming integrated.

The above discussion suggests that as the fraction of the minority population grows, the frequencies of encounters with similar and dissimilar neighbors, respectively, become more balanced, and the adaptive force dissipates, as observed in the minority density graph in figure 3, panel (b). The discussion also implies that both a longer tenure requirement and a larger probability of random relocation should make it easier for adaptation to dominate. This is observed in the graphs illustrating the marginal effects of the tenure requirement (panel (d)) and the fraction of random movers (panel (e)). The graphs also show that the length of tenure requirement and the amount of random relocation needed to obtain integration are not onerous. Five periods of tenure and five percent of the population randomly relocating generate a substantial integration, and ten percent random relocation is sufficient to generate a nearly full integration. Finally, panel (a) in the figure shows that segregation is generally inversely related to the population density. This is likely because it is easier to form isolated enclaves of similar individuals, separated by vacant locations, when the grid is sparsely populated.

3 Model with income and housing cost

As the previous section shows, when homophily is based on behavior and not on immutable identity, adaptation and integration, rather than segregation, appear to be the typical outcome. This result, however, assumes that there are no other economic or social forces influencing individuals' relocation decisions. In this section, we investigate how the tendency toward integration is affected by economic frictions. In particular, we extend the base model of Section 2 to incorporate income and housing rental cost. Individuals are now assumed to derive utility from both housing and (non-housing) consumption. Let c be the amount of income spent on consumption and π the utility from housing, which comes from the neighborhood interactions as defined in the base model. Individual i's overall utility is $u_i(c_i, \pi_i) = c_i^{\alpha}\pi_i^{1-\alpha}$, where $0 < \alpha < 1$. Parameter α controls the relative importance of consumption and housing in the individual's utility, with higher α making consumption relatively more important. If $\alpha = 0$, individuals only care about housing, as in the base model.

Each individual has income I_i, which is drawn randomly in the beginning of period 1. The individual receives that same income in every subsequent period. In particular, $I_i = \text{base income}_i \times \varepsilon_i$, where ε_i is lognormal with parameters $\mu = 0$ and $\sigma = 0.25$.[19] Thus, the mean and the standard deviation of income are:

---

[19] Lognormal distribution is one of several distributions that are commonly used to model income. As we discuss further in our supplement, Jasso (2023) shows that different distributions can generate different outcomes in the context of social distance and proportions integrationist and segregationist based on justice evaluation, and we hope to study the implications of using different distributions in future research.

$$\begin{aligned} \mathrm{E}\,[I_i] &= \text{base income}_i \times e^{\mu+\frac{\sigma^2}{2}} = \text{base income}_i \times 1.0317 \\ \mathrm{var}[I_i] &= \text{base income}_i^2 \times \left(e^{\sigma^2} - 1\right)(e^{2\mu+\sigma}) = \text{base income}_i^2 \times 0.0828. \end{aligned}$$

Individuals divide their income between consumption and housing rent. We assume that the rent is the same for every location in a district in a given period although the rents could differ across districts and across periods.

3.1 Model dynamics

As in the base model, individuals in the income-housing model start period 1 with initial values, $\tilde{\pi}_{i1}(a)$, $\tilde{\pi}_{i1}(b)$, $N_{i1}(a)$, and $N_{i1}(b)$. Their within-period behaviors are nearly identical to the base model, while relocation decisions have the additional considerations arising from housing costs.

Within-period behavior: In each period $t \geq 1$, individuals make their action choices based on their assessments, $\tilde{\pi}_{it}(a)$ and $\tilde{\pi}_{it}(b)$, play the coordination game with their neighbors using their choices, and receive housing utility π_{it}. In addition, they receive income I_i, pay rent, and spend the remainder on consumption. We assume that the individuals are not forced to pay more rent than their income so that consumption is non-negative. That is, letting $r_{\tau t}$ be the rent in district τ at period t, an individual living in that district consumes $c_{it} = I_i - \min\{r_{\tau t}, I_i\}$. Her overall utility in the period is $u_{it} = u_i(c_{it}, \pi_{it})$. At the end of the period, individuals update their assessments in the same way as in the base model.

Relocation behavior: Individuals can stay in their current location, move to another location in the current district, or move to a different district in the next period. As in the base model, we assume that individuals move when their housing utility falls below the happiness threshold level, provided that the tenure threshold is met.[20] For an individual wanting to relocate, let $\tilde{\pi}_{it+1}(\tau)$ be the housing utility she expects to receive in district τ, and let

$$\tilde{u}_{it+1}(\tau) = (I_i - \min\{r_{\tau t}, I_i\})^{\alpha}\tilde{\pi}_{it+1}(\tau)^{1-\alpha}$$

be the overall utility she expects to receive from moving to the district in the next period.[21] The individual will prefer district L over district R if $\tilde{u}_{it+1}(L) > \tilde{u}_{it+1}(R)$. That is,

$$(I_i - \min\{r_{Lt}, I_i\})^{\alpha}\tilde{\pi}_{it+1}(L)^{1-\alpha} > (I_i - \min\{r_{Rt}, I_i\})^{\alpha}\tilde{\pi}_{it+1}(R)^{1-\alpha}. \tag{1}$$

Rearranging yields,

---

[20] The decision to relocate is based on the individual's current housing utility rather than her overall utility, which makes the comparison of the segregation outcome with the base model more meaningful. This assumption may be justified by the fact that there are uncertainties about the new neighborhood which is resolved only after the individual settles there.

[21] We implicitly assume that individuals are boundedly rational in that they take the current rents as the rents that will prevail in the next period rather than trying to forecast them.

$$\min\{r_{Lt},\, I_i\} < I_i - \left(\frac{(I_i - \min\{r_{Rt},\, I_i\})^{\alpha}\tilde{\pi}_{it+1}(R)^{1-\alpha}}{\tilde{\pi}_{it+1}(L)^{1-\alpha}}\right)^{\frac{1}{\alpha}}. \tag{2}$$

Thus, the right-hand side of inequality (2) is the individual's maximum willingness to pay to rent a house in district L. If inequality (1) is reversed, the individual will prefer to move to district R, and her willingness to pay can be derived analogously. If inequality (1) holds with equality, the individual is indifferent between the two districts, and she is assumed to choose one with equal probability. For individuals who are happy in their current location and wish to remain there, their maximum willingness to pay is assumed to be their income.

After the location preference and the willingness to pay have been determined for all the individuals, their actual locations and the rents in the next period are determined in the following way. If the demand for housing is greater than or equal to the supply of housing in a district, the rent in that district is the "market clearing price." That is, if the number of individuals wanting to stay in the district and those wanting to move there from the other district is greater than or equal to the number of locations in the district, $N^2/2$, the rent is the $N^2/2$-*th* highest willingness to pay, and the individuals with willingness to pay greater than or equal to the rent will be located in the district, with those wishing to stay in their current location in the district remaining there and the others being assigned to a random location in the district. If there are ties, individuals are chosen with equal probability as needed. For the district in which the supply of housing exceeds demand, individuals desiring to live there plus those who have been priced out of the other district, that is, those who could not live in the other district because their willingness to pay were too low, will be located in this district. The rent in this district is the lowest willingness to pay among the individuals who initially wished to live in the district. Thus, the model implicitly assumes that individuals who are priced out of their first-choice district simply pay the "going rent" in the other district.

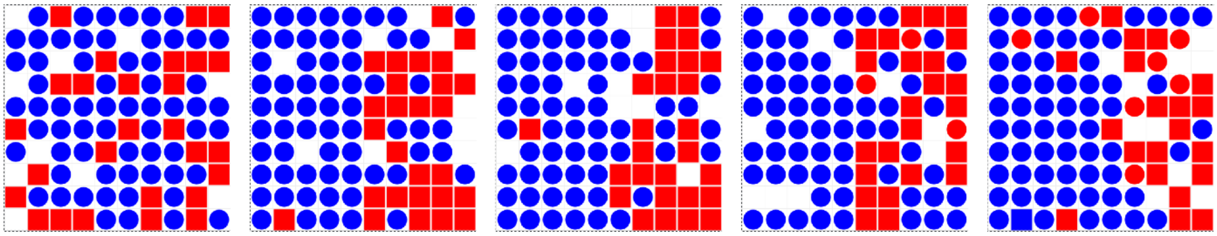
3.2 Simulation and results

For the simulations, we set $\alpha = 1/2$ so that consumption and housing weigh equally in individuals' utility. Because income constrains where an individual can live when housing is costly, a larger income disparity between the majority and the minority groups should have a greater effect on segregation. To verify, we fixed the majority base income at 10 and varied the minority base income, as a fraction of the majority base income, while keeping the remaining parameters constant at the same values as in Section 2 ($d = 0.9, m = 0.3, \bar{u} = 8, \bar{t} = 5$, and $p = 0.1$). The result is given in figure 5(f), which shows that segregation generally increases as the minority income falls. Since our aim is to explore the effects of income inequality and housing cost on segregation, we set the minority base income at 40% of the majority base income for the remainder of the simulations.[22] The simulation result for the income-housing model with these incomes and the remaining parameters and the initial grid

---

[22] The graph of the majority and the minority income distributions under this parameter value are provided in the supplement to this paper.

configuration set identical to Section 2 is given in figure 4(a). In contrast to the base model, segregation remains significant in the income-housing model even at period 1000. The dissimilarity and the exposure indices for the simulation as well as the average indices for 100 simulations at those parameters are given panel (b) and panel(c).

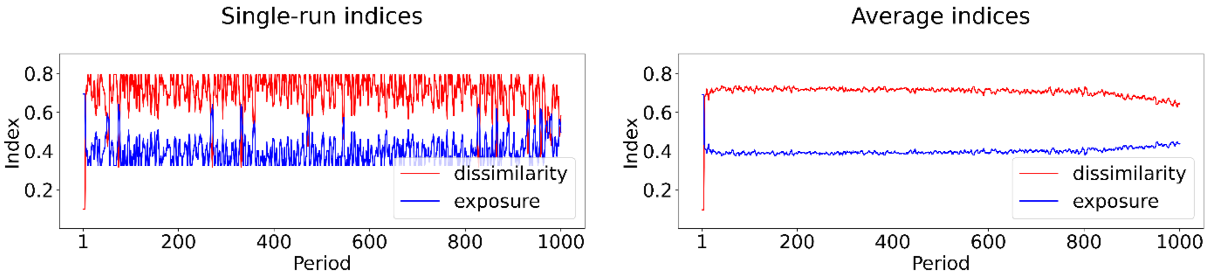


$t = 1$ $t = 10$ $t = 100$ $t = 500$ $t = 1000$

(a) Grid configuration: single run.

	t = 1	10	100	500	1000
dissimilarity: single	0.101	0.709	0.709	0.683	0.582
dissimilarity: average	0.097	0.691	0.721	0.711	0.646
exposure: single	0.694	0.403	0.403	0.426	0.498
exposure: average	0.691	0.407	0.390	0.396	0.437

(b) Dissimilarity and exposure indices.



(c) Dissimilarity and exposure indices.

Figure 4: Income-housing model.

To see how robust segregation is in the income-housing model, we varied the parameters one at a time and compared the results with the base model. Figure 5 gives the two segregation indices, averaged over 100 simulations. As the figure shows, incorporating income inequality and housing cost typically leads to higher segregation, and the difference appear substantial for many parameter values.

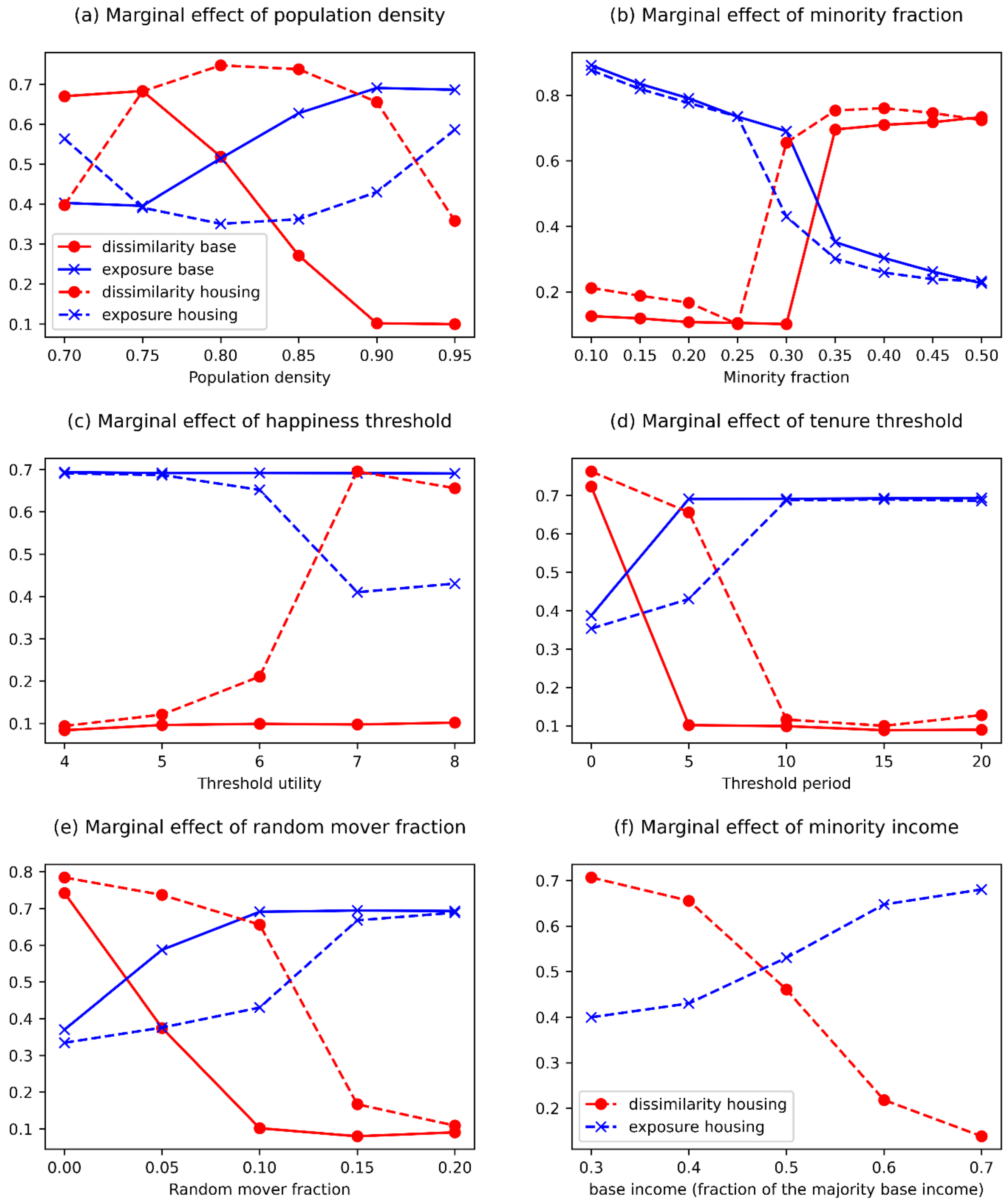


Figure 5: Marginal effects on segregation, income-housing model.

3.3 Discussion

The simulation results show that economic friction in the form of housing cost and income disparity between the groups generally impedes integration. The initial difference in the behavioral norms of the two groups means that both groups start out preferring to be among themselves. Since there are more majority individuals than the minority individuals, the demand for housing is higher for the majority dominated district. The differential demands drive up the housing price of the majority district relative to the minority concentrated district and discourages the poor majority individuals and the minority individuals who behave like the majority from moving into the majority district. This is an exclusionary process similar to the earlier models with wealth and status-based homophily. Unlike those models however our individuals would prefer to move to a lower priced district and enjoy higher consumption, all else being equal, since there is no inherent advantage to living in a more expensive district. Nevertheless, because the economic forces concentrate the minorities into one district, their behaviors reinforce each other, and their initial modal behavior tends to survive longer. This in turn makes the minority district less attractive to the majority individuals despite the lower housing cost. Thus, the population end up more segregated than when these economic forces are absent.

3.4 Empirical link

To link our model to the observed residential segregation in the U.S., we collected county-level segregation, income, and housing price data from the Federal Reserve Economic Data database for years 2009 to 2020. We then estimated the fixed effect model:

$$\begin{aligned} \mathit{segregation}_{\mathrm{it}} = {} & \beta_0 + \beta_1\ \mathit{inequality}_{\mathrm{it}} + \beta_2\ \mathit{housing\ price}_{\mathrm{it}} + \beta_3\ \mathit{income}_{\mathrm{it}} \\ & + \theta_1\ \mathit{inequality}_{\mathrm{it}} \times \mathit{housing\ price}_{\mathrm{it}} + \theta_2\ \mathit{inequality}_{\mathrm{it}} \times \mathit{housing\ price}_{\mathrm{it}} \\ & + \theta_3\ \mathit{housing\ price}_{it} \times \mathit{income}_{it} + \mathit{population}_i + \gamma_i + \delta_t + \epsilon_{it}, \end{aligned}$$

where *segregation* is the white to non-white dissimilarity index measured as a percentage, *inequality* is the ratio of the average income of the top 20% earners to the bottom 20%, *housing price* is the housing price index, *income* is the inflation-adjusted median income, and *population* is the number of people.[23] The results of the regression with and without the interacting terms are given in table 1. In both regressions, income inequality and housing price increase segregation in conformance with our model. Although the effect of housing price is statistically insignificant in the baseline regression, it is likely due to interference by the income effect, which has the opposite sign in that regression. Indeed, as is typical in such cases, it becomes significant once the interaction with the income is controlled for. Inclusion of the interaction terms also clarifies that it is income inequality rather than income level that drives segregation. As seen in column 3, once the level of inequality is controlled for, income has no direct effect on the level of segregation although it has a small mitigating effect on the effect of housing price.

---

[23] The quantile ratio index, rather than the Gini coefficient, is used as a measure of inequality because FRED reports only the quantile ratio at the county level.

Table 1: Effects of inequality, housing price, and income on segregation, 2009-2020.

	Fixed Effect		With Interaction Terms	
	Coefficient	Std. err.	Coefficient	Std. err
inequality	0.0821**	0.0362	0.1816*	0.1065
housing price	0.0033	0.0054	0.0353**	0.0157
income ($1000)	-0.0428**	0.0205	0.019	0.0517
population (1000 people)	0.0101*	5.34e-03	0.0107*	5.48e-03
Inequality × housing price	-	-	-0.0008	0.0006
Inequality × income	-	-	4.24e-07	3.18e-06
Housing price × income	-	-	-3.72e-07*	2.08e-07
Number of observations	26,255		26,255	
R^2 (overall)	0.1112		0.1044	

Notes: County and year fixed effects are controlled for. Robust standard errors are reported and clustered at the county level.

References

[1] Berg, N., Hoffrage, U., & Abramczuk, K. (2010). Fast Acceptance by Common Experience: FACE-recognition in Schelling's model of neighborhood segregation. *Judgment and Decision Making*, 5:391-410.

[2] Bernard, S. & Willer, R. (2007). A Wealth and Status-Based Model of Residential Segregation. *The Journal of Mathematical Sociology,* 31(2):149-174.

[3] Charles, C. Z. (2003). The dynamics of racial residential segregation. *Annual Review of Sociology*, 29:167-207.

[4] Clark, W. A. V. & Fossett, M. (2008). Understanding the social context of the Schelling segregation model. *Proceedings of the National Academy of Sciences of the United States of America,* 105:4109-4114.

[5] Conley, J. & Neilson, W. S. (2013) Endogenous coordination and discoordination games: Multiculturalism and assimilation*.* *Journal of Economic Behavior and Organization*, 92:176-191.

[6] Crowell, A. & Fossett, M. (2022). Metropolitan racial residential segregation in the United States: A microlevel and cross-context analysis of Black, Latino, and Asian segregation. *Demographic Research,* 46:217-260.

[7] Föllmer, H. (1974). Random Economies with Many Interacting Agents. *Journal of Mathematical Economics,* 1:51-62.

[8] Föllmer, H, Horst, U, and Kirman, A. (2005). Equilibria in financial markets with heterogeneous agents: a probabilistic perspective. *Journal of Mathematical Economics,* 41:123-155.

[9] Fossett, M. (2011). Generative Models of Segregation: Investigating Model Generated Patterns of Residential Segregation by Ethnicity and Socioeconomic Status. *The Journal of Mathematical Sociology,* 35:114-145.

[10] Fu, F., Nowak, M. A., Christakis, N. A., & Fowler, J. H. (2012). The evolution of homophily. *Scientific Reports,* 2:845.

[11] Jasso, G. (2010). Linking Individuals and Societies. *Journal of Mathematical Sociology,* 34:1-51.

[12] Jasso, G. (2023). Fifty Years of Justice Research: Seven Signposts Past and Future. *Social Justice Research*, 36:305-324.

[13] Li, S., Chang, K. L., & Wang, L. (2020). Racial residential segregation in multiple neighborhood markets: a dynamic sorting study. *Journal of Economic Interaction and Coordination,* 15:363-383.

[14] Malmberg, B. & Clark, W. A. V. (2020). Migration and Neighborhood Change in Sweden: The Interaction of Ethnic Choice and Income Constraints. *Geographical Analysis,* 53:259-282.

[15] Pancs, R. & Vriend, N. (2007). Schelling's spatial proximity model of segregation revisited. *Journal of Public Economics*, 91:1-24.

[16] Schelling, T. C. (1971). Dynamic Models of Segregation. *Journal of Mathematical Sociology,* 1:143-186.

[17] Sethi, R. & Somanathan, R. (2004). Inequality and Segregation. *Journal of Political Economy,* 112(6): 1296-1321.

[18] Vinkovic, D & Kirman, A. (2006). A physical analogue of the Schelling model. *Proceedings of the National Academy of Sciences of the United States of America,* 103: 19261-19265.

[19] Zhang, J. (2004). A Dynamic Model of Residential Segregation. *Journal of Mathematical Sociology,* 28:147-170.

[20] Zhang, J. (2009). Tipping and Residential Segregation: A Unified Schelling Model. *Journal of Regional Science,* 51:167-193.