

Intermediate Microeconomics

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Problem Set 3: suggested solutions

1. Consider an individual whose utility function over money is $u(w) = 1 + 2w^{\frac{1}{2}}$.

- (a) Determine the individual's attitude toward risk. Does it depend on w ?

Solution: We have

$$\begin{aligned}u'(w) &= w^{-\frac{1}{2}}, \quad \text{and} \\u''(w) &= -\frac{1}{2}w^{-\frac{3}{2}} < 0 \quad \text{for all } w \text{ (except } w = 0\text{)}.\end{aligned}$$

So, the individual is strictly risk-averse at all wealth level.

- (b) Suppose the individual has initial wealth $\text{¥}W$ and faces the possible loss of $\text{¥}\frac{W}{2}$. The probability that the loss will occur is $\frac{1}{2}$. Suppose insurance is available at price p , where p is not necessarily the fair price. Find the optimal amount of insurance the individual should buy. You may assume that the solution is interior.

Solution: The individual solves

$$\begin{aligned}&\max_x \frac{1}{2}u(W - px) + \frac{1}{2}u\left(W - \frac{W}{2} - px + x\right) \\&\iff \max_x \frac{1}{2}\left(2(W - px)^{\frac{1}{2}}\right) + \frac{1}{2}\left(2\left(\frac{W}{2} + (1 - p)x\right)^{\frac{1}{2}}\right).\end{aligned}$$

Assuming $p < 1$, FOC is given by

$$\begin{aligned}&\frac{1}{2}(W - px)^{-\frac{1}{2}}(-p) + \frac{1}{2}\left(\frac{W}{2} + (1 - p)x\right)^{-\frac{1}{2}}(1 - p) = 0. \\&(1 - p)^{-2}\left(\frac{W}{2} + (1 - p)x\right) = p^{-2}(W - px) \\&(1 - p)^{-2}\left(\frac{W}{2}\right) + (1 - p)^{-1}x = p^{-2}W - p^{-1}x \\&(1 - p)^{-1}x + p^{-1}x = Wp^{-2} - \left(\frac{W}{2}\right)(1 - p)^{-2}. \\&\implies x^* = \frac{Wp^{-2} - \left(\frac{W}{2}\right)(1 - p)^{-2}}{(1 - p)^{-1} + p^{-1}} = W\left(\frac{\frac{1-p}{p} - \frac{p}{2(1-p)}}{(1 - p) + p}\right) \\&= W\left(\frac{1 - p}{p} - \frac{p}{2(1 - p)}\right) = \frac{W(p^2 - 4p + 2)}{2p(1 - p)}.\end{aligned}$$

- (c) Is there a price at which the individual will not want to buy any insurance? If so, find it. If no, explain.

Solution: Using the quadratic formula (and assuming $p < 1$), we obtain

$$x^* = \frac{W(p^2 - 4p + 2)}{2p(1 - p)} \leq 0 \implies p \geq 2 - \sqrt{2} \approx 0.5858.$$

2. Consider an investor whose utility function over money is $u(w) = w^a$, where $0 < a < 1$. The investor can invest in a riskless asset that returns 1 (gross return per \$1 invested) for sure, or a risky asset that returns (gross) 1.1 with probability $\frac{3}{5}$ and 0.9 with probability $\frac{2}{5}$.

- (a) Find the investor's Arrow-Pratt measure of absolute risk aversion. Is the investor risk averse, risk neutral, or risk loving? How does her degree of absolute risk aversion change as her wealth changes?

Solution: We have,

$$\begin{aligned} u'(w) &= aw^{a-1} \quad \text{and} \quad u''(w) = a(a-1)w^{a-2} \\ \implies R_A(w) &= -\frac{u''(w)}{u'(w)} = -\frac{a(a-1)w^{a-2}}{aw^{a-1}} = \frac{(1-a)}{w} > 0 \implies \text{strictly risk averse.} \\ \frac{dR_A(w)}{dw} &= -(1-a)w^{-2} < 0 \implies \text{DARA.} \end{aligned}$$

- (b) Suppose the investor's initial wealth is \$W\$. Find the optimal amount to invest in the risky asset (use x to denote the amount invested in the risky asset).

Solution: Let $g(x)$ be the lottery that results from investing \$x in the risky asset. We have

$$g(x) = \begin{cases} W - x + 1.1x = W + 0.1x & \text{with probability } \frac{3}{5} \\ W - x + 0.9x = W - 0.1x & \text{with probability } \frac{2}{5} \end{cases}$$

So the investor solves:

$$\max_x U(g(x)) \iff \max_x \frac{3}{5}(W + 0.1x)^a + \frac{2}{5}(W - 0.1x)^a$$

Solving FOC yields,

$$\begin{aligned} \left(\frac{3a(0.1)}{5}\right)(W + 0.1x)^{a-1} - \left(\frac{2a(0.1)}{5}\right)(W - 0.1x)^{a-1} &= 0 \\ \implies 3(W + 0.1x)^{a-1} &= 2(W - 0.1x)^{a-1} \\ W + 0.1x &= \left(\frac{3}{2}\right)^{\frac{1}{1-a}}(W - 0.1x) \\ \left(\left(\frac{3}{2}\right)^{\frac{1}{1-a}} + 1\right)(0.1x) &= \left(\left(\frac{3}{2}\right)^{\frac{1}{1-a}} - 1\right)W \\ x^* &= \frac{\left(\left(\frac{3}{2}\right)^{\frac{1}{1-a}} - 1\right)10W}{\left(\frac{3}{2}\right)^{\frac{1}{1-a}} + 1} \end{aligned}$$

- (c) How does the amount invested in the risky asset changes as the investor's wealth increases? Give a brief explanation of this result (in light of part (a)).

Solution: It increases since $\frac{dx^*}{dW} > 0$. Since the investor has a DARA utility function, the individual becomes less risk averse as her wealth increases. Therefore she takes on more risk by putting greater amount of her wealth into the risky asset.

3. Suppose a firm's production function is given by $f(L, K) = L^a K^b$, where $a > 0$ and $b > 0$. As usual, let w , r , and p be the price of labor, capital, and the output good, respectively.

- (a) A firm has a constant returns to scale if $f(\lambda L, \lambda K) = \lambda f(L, K)$ for all $\lambda > 0$. It has a decreasing returns to scale if $f(\lambda L, \lambda K) < \lambda f(L, K)$ and an increasing returns to scale if $f(\lambda L, \lambda K) > \lambda f(L, K)$ for all $\lambda > 1$, respectively. Determine the conditions on a and b under which the firm has constant, decreasing, and increasing returns to scale, respectively.

Solution: We have $f(\lambda L, \lambda K) = (\lambda L)^a (\lambda K)^b = \lambda^{a+b} L^a K^b = \lambda^{a+b} f(L, K)$. So, the firm exhibits DRS if $a+b < 1$, CRS if $a+b = 1$, and IRS if $a+b > 1$.

- (b) Find the firm's (long-run) cost function.

Solution: "MRTS = price ratio" and output equation yield,

$$\frac{\frac{\partial f}{\partial L}}{\frac{\partial f}{\partial K}} = \frac{aL^{a-1}K^b}{bL^aK^{b-1}} = \frac{aK}{bL} = \frac{w}{r} \implies K = \frac{bw}{ar}L \quad \text{and} \quad L^a \left(\frac{bw}{ar}L \right)^b = q.$$

Thus,

$$\begin{aligned} L^* &= \left(\frac{ar}{bw} \right)^{\frac{b}{a+b}} q^{\frac{1}{a+b}} \implies K^* = \left(\frac{bw}{ar} \right) \left(\frac{ar}{bw} \right)^{\frac{b}{a+b}} q^{\frac{1}{a+b}} = \left(\frac{bw}{ar} \right)^{\frac{a}{a+b}} q^{\frac{1}{a+b}} \\ \implies c(q) &= w \left(\frac{ar}{bw} \right)^{\frac{b}{a+b}} q^{\frac{1}{a+b}} + r \left(\frac{bw}{ar} \right)^{\frac{a}{a+b}} q^{\frac{1}{a+b}} \\ &= \left[\left(\frac{a}{b} \right)^{\frac{b}{a+b}} w^{\frac{a}{a+b}} r^{\frac{b}{a+b}} + \left(\frac{b}{a} \right)^{\frac{a}{a+b}} w^{\frac{a}{a+b}} r^{\frac{b}{a+b}} \right] q^{\frac{1}{a+b}} \\ &= \left[\left(\frac{a}{b} \right)^{\frac{b}{a+b}} + \left(\frac{b}{a} \right)^{\frac{a}{a+b}} \right] w^{\frac{a}{a+b}} r^{\frac{b}{a+b}} q^{\frac{1}{a+b}}. \end{aligned}$$

- (c) Suppose a and b are such that the firm has decreasing returns to scale. Is the cost function concave, linear, or convex function of q (output quantity)? Find the firm's (long-run) supply function.

Solution: To make the writing concise, let $\gamma = \left[\left(\frac{a}{b} \right)^{\frac{b}{a+b}} + \left(\frac{b}{a} \right)^{\frac{a}{a+b}} \right] w^{\frac{a}{a+b}} r^{\frac{b}{a+b}}$ be the "constant" part of the cost function for the remainder of this question so that $c(q) = \gamma q^{\frac{1}{a+b}}$. If $a+b < 1$, then $\frac{1}{a+b} > 1$, which means

$c''(q) = \left(\frac{\gamma}{a+b}\right) \left(\frac{1}{a+b} - 1\right) q^{\frac{1}{a+b}-2} > 0$ (except possibly at $q = 0$). So, $c(q)$ is a strictly convex function of q . Solving $p = MC(q)$ yields

$$p = \left(\frac{\gamma}{a+b}\right) q^{\frac{1}{a+b}-1} = \left(\frac{\gamma}{a+b}\right) q^{\frac{1-(a+b)}{a+b}} \implies q^* = \left(\frac{(a+b)p}{\gamma}\right)^{\frac{a+b}{1-(a+b)}}.$$

Note that the second order condition is satisfied since $c(q)$ is strictly convex.

- (d) Suppose a and b are such that the firm has increasing returns to scale. Is the cost function concave, linear, or convex function of q ? What is the firm's (long-run) profit maximizing output level?

Solution: By above, $c''(q) < 0$ when $a + b > 1$, so the cost function is linear. Since MC is always decreasing the second order condition cannot be satisfied, and the firm always wants to increase its production. That is, $q^* = \infty$ (or, technically, π -maximizing output level does not exist).

- (e) Suppose a and b are such that the firm has constant returns to scale. Is the cost function concave, linear, or convex function of q ? Find the firm's (long-run) supply function. What does this say about the shape of firm's (long-run) supply curve?

Solution: By above, $c''(q) = 0$ when $a + b = 1$, so marginal cost is constant and the cost function is linear. Solving $p = MC(q)$ in this case yields $p = \frac{\gamma}{a+b}$. Note that $\frac{\gamma}{a+b}$ is also the minimum MC (since MC is constant). Thus,

$$q^* = \begin{cases} \infty \text{ (}\pi\text{-max soln does not exist)} & \text{if } p > \frac{\gamma}{a+b} \\ \text{any output } \geq 0 & \text{if } p = \frac{\gamma}{a+b} \\ 0 \text{ (shuts down)} & \text{if } p < \frac{\gamma}{a+b}. \end{cases}$$

Thus, the supply curve is flat at $p = \frac{\gamma}{a+b}$.

4. Consider a market where the supply and the demand are given by

$$Q^S(P) = 100P \quad \text{and} \quad Q^D(P) = 2000 - 100P.$$

- (a) Find the equilibrium price, quantity, consumer surplus, producer surplus, and the aggregate surplus.

Solution:

$$\begin{aligned} Q^S(P) &= 100P = 2000 - 100P = Q^D(P) \\ \implies P^* &= 10 \quad \text{and} \quad Q^* = 100(10) = 1,000. \end{aligned}$$

The surpluses are

$$CS = \frac{1}{2}(20 - 10)(1000) = 5,000$$

$$PS = \frac{1}{2}(10 - 0)(1000) = 5,000$$

$$AS = 10,000.$$

- (b) Suppose the government wants to raise revenue by imposing tax of ¥4 per unit. What is the price producers get, the price consumers pay, the equilibrium quantity, the tax revenue, and the dead weight loss?

Solution: We now set

$$\begin{aligned}Q^S(P) &= 100(P^S) = 2000 - 100(P^S + t) = Q^D(P) \\200P^S &= 2000 - 100t\end{aligned}$$

To obtain

$$\begin{aligned}P^S &= 10 - \frac{t}{2} = 8 \\P^D &= 8 + 4 = 12 \\Q^T &= 800 \\GR &= 4(800) = 3,200 \\DWL &= (0.5)(t)(Q^* - Q^T) = (0.5)(4)(1000 - 800) = 400.\end{aligned}$$

- (c) Suppose the government is thinking about imposing an ad valorem tax instead of per unit tax. What does the tax rate has to be to keep the price consumers pay the same as in the per unit tax case?

Solution: Setting

$$Q^S(P) = 100(P^S) = 2000 - 100(P^S + tP^S) = Q^D(P)$$

yields

$$200P^S + 100tP^S = 2000 \quad \Rightarrow \quad P^S = \frac{2000}{200 + 100t}$$

We want

$$\begin{aligned}(1 + t)P^S &= 12 \quad \Rightarrow \quad (1 + t)\frac{2000}{200 + 100t} = 12 \\&\Rightarrow \quad 2000 + 2000t = 2400 + 1200t \\&\Rightarrow \quad t = 50\%\end{aligned}$$

We have

$$\begin{aligned}P^S &= \frac{2000}{200 + 100(.5)} = 8 \\P^T &= (1 + 0.5)P^S = 12\end{aligned}$$

- (d) Which tax scheme is better for the economy? Why?

Solution: Note that even in valorem case, $Q^T = 100P^S = 800$. Since this means that CS , PS , GR , and DWL are all the same under both tax schemes. So they are equivalent.

5. Consider again the market where the supply and the demand are given by

$$Q^S(P) = 100P \quad \text{and} \quad Q^D(P) = 2000 - 100P.$$

- (a) Suppose the government imposes a price floor of $P_f = 12$. Assuming that the government buys any excess supply, find the consumer surplus, producer surplus, government revenue, and aggregate surplus. What is the deadweight loss (in comparison to the no intervention case)?

Solution: Since $P_f = 12 > 10 = P^*$, the price floor is binding. The quantity demanded and supplied at this price is given by

$$Q^S(P_f) = 100P_f = 100(12) = 1200$$

$$Q^D(P_f) = 2000 - 100(P_f) = 2000 - 100(12) = 800$$

$$\text{Excess supply} = Q^S(P_f) - Q^D(P_f) = 1200 - 800 = 400.$$

The relevant surpluses are

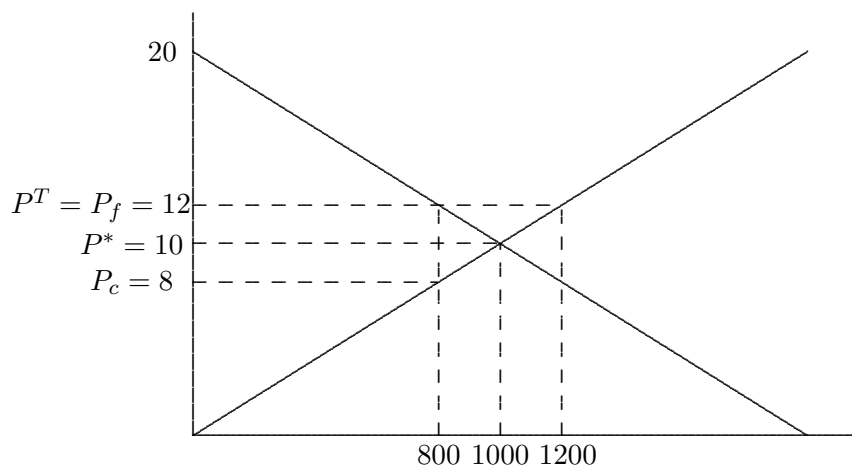
$$CS' = \frac{1}{2}(20 - P_f) \times Q^D(P_f) = \frac{1}{2}(20 - 12)(800) = 3200$$

$$PS' = \frac{1}{2}(P_f - 0) \times Q^S(P_f) = \frac{1}{2}(12 - 0)(1200) = 7200$$

$$GR' = -P_f \times \text{excess supply} = 12(400) = -4800$$

$$AS' = CS' + PS' + GR' = 3200 + 7200 - 4800 = 5600$$

$$DWL = AS - AS' = 10000 - 5600 = 4400.$$



- (b) Suppose the government imposes a price ceiling $p_c = 8$. What is the change in consumer surplus from the no intervention case?

Solution: Since $P_c = 8 < 10 = P^*$, the price ceiling is binding. This means that there will be a shortage. That is, the quantity transacted will be whatever the producers are willing to supply at price P_c . That is $Q^T = Q^S(P_c) = 100(8) = 800$. The price that the consumers are willing to pay for Q^T is given by

$$Q^T = Q^D(P^T) \iff 800 = 2000 - 100(P^T) \iff P^T = 12.$$

Thus,

$$\begin{aligned} CS' &= \frac{1}{2}(20 - P^T) \times Q^T + (P^T - P_c) \times Q^T \\ &= \frac{1}{2}(20 - 12)(800) + (12 - 8)(800) = 3200 + 3200 = 6400 \end{aligned}$$

$$\implies CS' - CS = 6400 - 5000 = 1400.$$

- (c) In general (i.e., do not restrict attention to the supply and the demand functions given above), will consumers always benefit from price ceiling? (A carefully drawn graphical example will suffice).

Solution: When demand curve is relatively steep (not price sensitive) and supply curve is relatively flat (price sensitive), then price ceiling can actually reduce consumer surplus. In the picture below, consumer surplus without intervention is $CS = A + B$. When price ceiling P_c is imposed, the new consumer surplus is $CS' = A + C$. In the picture, the difference $CS' - CS = C - B$ is negative.

