

Intermediate Microeconomics

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Problem Set 2: suggested solutions

1. Consider a consumer with income $I = 10$ and utility function:

$$u(x_1, x_2) = x_1 + 2x_2^{\frac{1}{2}}.$$

- (a) Suppose prices are $p_1 = 2$ and $p_2 = 1$. Find the consumer's demand.

Solution: We have

$$MRS = \frac{1}{x_2^{-\frac{1}{2}}} = x_2^{\frac{1}{2}} = \frac{p_1}{p_2} \Rightarrow x_2 = \frac{p_1^2}{p_2^2} = \frac{4}{1} = 4.$$

and

$$p_1x_1 + p_2x_2 = I \Rightarrow x_1 = \frac{I - p_2x_2}{p_1} = \frac{I - \frac{p_1^2}{p_2}}{p_1} = \frac{10 - 4}{2} = 3$$

- (b) Suppose now that the prices are $p'_1 = 4$ and $p_2 = 1$. Find the consumer's demand.

Solution: Now, we have

$$MRS = \frac{1}{x_2^{-\frac{1}{2}}} = x_2^{\frac{1}{2}} = \frac{p'_1}{p_2} \Rightarrow x_2 = \frac{p'^2_1}{p^2_2} = \frac{16}{1} = 16.$$

and

$$p'_1x_1 + p_2x_2 = I \Rightarrow x_1 = \frac{I - p_2x_2}{p'_1} = \frac{I - \frac{p'^2_1}{p_2}}{p'_1} = \frac{10 - 16}{4} < 0$$

But since consumption cannot be negative, this means that $x_1 = 0$ (i.e., there are no interior solution). This forces $x_2 = \frac{I}{p_2} = 10$.

- (c) Interpret this result.

Solution At the solution, $x_1 = 0$ and $x_2 = 10$, we have

$$\frac{MU_1}{p'_1} = \frac{1}{p'_1} = \frac{1}{4} < .316 \approx (10)^{-\frac{1}{2}} = \frac{x_2^{-\frac{1}{2}}}{p_2} = \frac{MU_2}{p_2}.$$

So, at this consumption bundle, the consumer would like to reduce consumption of good 1 further in favor of good 2. However, this is not possible since the consumer is already consuming zero units of good 1.

2. A consumer who lives for two periods has a standard Cobb-Douglas utility function:

$$u(c_1, c_2) = c_1^\alpha c_2^\beta,$$

where c_t = consumption in period t and $\alpha + \beta = 1$. Her income in period one is $I_1 = 500$ and $I_2 = 400$, and she can lend or borrow at interest rate $r = 0.2$.

- (a) Find the optimal consumption demand.

Solution: The budget equation is

$$\begin{aligned}(1+r)c_1 + c_2 &= (1+r)I_1 + I_2 \\ (1.2)c_1 + c_2 &= (1.2)500 + 400 = 1000.\end{aligned}$$

So the demand functions are:

$$\begin{aligned}c_1 &= \alpha \left(\frac{(1+r)I_1 + I_2}{1+r} \right) = \alpha \left(\frac{1000}{1.2} \right) \approx \alpha(833.33) \\ c_2 &= \beta \left(\frac{(1+r)I_1 + I_2}{1} \right) = \beta(1000).\end{aligned}$$

- (b) What values of α , if any, makes the consumer a borrower? Interpret this result.

Solution: For the consumer to be a borrower, we need

$$\alpha(833.33) = c_1 > I_1 = 500 \quad \Rightarrow \quad \alpha > \frac{500}{833.33} \approx 0.60.$$

The exponents α and β represent the relative contributions of consumption in the two periods to the consumer's overall utility. Higher α means period 1 consumption is relatively more important, which makes the consumer more likely to borrow from the future to consume more in period 1.

- (c) Suppose now that $\alpha = \frac{4}{7}$ but that r is no longer 0.2. What values of r , if any, makes the consumer a borrower? Interpret this result.

Solution: For the consumer to be a borrower, we need

$$\begin{aligned}\alpha \left(\frac{(1+r)I_1 + I_2}{1+r} \right) &= \alpha \left(I_1 + \frac{I_2}{1+r} \right) = c_1 > I_1 = 500 \\ \Rightarrow \quad \frac{400}{1+r} &> \frac{7}{4}(500) - 500 \quad \Rightarrow \quad 400 > 375(1+r) \\ \Rightarrow \quad r &< \frac{25}{375} = 0.0667\end{aligned}$$

The interest rate r is the cost of borrowing. So, lower interest rate makes borrowing more likely.

3. Let $u(x) = \min \{2x_1, x_2\}$.

(a) Find the Hicksian demand.

Solution: Expenditure minimization problem is

$$\min_{x_1, x_2} p_1 x_1 + p_2 x_2 \quad \text{s.t.} \quad \min \{2x_1, x_2\} = u.$$

The solution occurs where

$$(1) \ 2x_1 = x_2 \quad \text{and} \quad (2) \ \min \{2x_1, x_2\} = u.$$

Thus,

$$x_1^h(p_1, p_2, u) = \frac{u}{2} \quad \text{and} \quad x_2^h(p_1, p_2, u) = u.$$

(b) Expenditure function $e(p_1, p_2, u)$ is the minimum amount of money needed to achieve utility level u when prices are p_1 and p_2 . Find the expenditure function this utility function.

Solution:

$$e(p_1, p_2, u) = p_1 x_1^h(p_1, p_2, u) + p_2 x_2^h(p_1, p_2, u) = \frac{p_1 u}{2} + p_2 u.$$

4. Consider a leisure-consumption model in which an individual is deciding her daily consumption of leisure and the composite consumption good. Her utility is $u(L, Y) = a \ln L + Y$, where L and Y denote the amount of leisure and the composite good, respectively. The individual has 24 hours of time in total. The hourly wage is w , and the price of the consumption good is p .

(a) Find the individual's (Marshallian) demand for leisure and the consumption good.

Solution: Solving the first order condition for utility maximization yields,

$$MRS = \frac{\frac{\partial u}{\partial L}}{\frac{\partial u}{\partial Y}} = \frac{a}{L} = \frac{w}{p} \implies L^* = \frac{ap}{w} \text{ and } Y^* = \frac{24w - wL^*}{p} = \frac{24w - ap}{p}.$$

(b) For what values of a , if any, will the individual not work? Explain this result.

Solution: The individual will not work if $L^* = \frac{ap}{w} \geq 24 \iff a \geq \frac{24w}{p}$. Note that as a increases, the individual's value for leisure relative to the consumption good increases; therefore, her incentive to work decreases, holding everything else constant. Indeed, if $a \geq \frac{24w}{p}$ then her MRS_{LY} at 24 hours of leisure is $\frac{a}{L} \geq \frac{24w}{24p} = \frac{w}{p}$. If the weak inequality holds with equality then that means 24 hours of leisure (zero hours of work) is optimal. If it holds with strict inequality, then it means that the (relative) market price of leisure is cheaper than her (relative) valuation for leisure. Thus, she in fact wants to buy more leisure, but she cannot because she's already consuming the maximum amount of leisure possible.

- (c) Suppose the hourly wage increases from $w = 2$ to $w' = 4$ while the price of the composite good remains fixed at $p = 1$. Find the substitution, the income, and the total effects on leisure.

Solution: For this utility function, the Marshallian demand and the Hicksian demands for leisure is determined solely by the “MRS = price ratio” condition. Therefore, they are the same. Thus, assuming interior solution and letting u^0 denote the utility at the original Marshallian demand, $L(w, I)$ and $Y(w, I)$, we have:

$$\begin{aligned} TE_L &= x_L(w', I) - x_L(w, I) = \frac{a}{4} - \frac{a}{2} = -\frac{a}{4} \\ SE_L &= h_L(w', u^0) - x_L(w, I) = \frac{a}{4} - \frac{a}{2} = -\frac{a}{4} \\ \implies IE_L &= TE_L - SE_L = 0. \end{aligned}$$

5. Consider an individual whose utility function over money is $u(w) = w^{\frac{1}{2}}$. The individual is facing a risk of losing ¥100 with probability $\frac{1}{2}$ and nothing with probability $\frac{1}{2}$.

- (a) Suppose the individual’s current wealth is ¥1,000. What is the maximum amount of money that she is willing to pay to avoid this risk?

Solution: The maximum amount of money she is willing to pay, M , must solve

$$\begin{aligned} u(1000 - M) &= \frac{1}{2}u(1000 - 100) + \frac{1}{2}u(1000 - 0) \\ \sqrt{1000 - M} &= \frac{1}{2}\sqrt{900} + \frac{1}{2}\sqrt{1000} \approx 30.81 \\ M &= 1000 - 949.34 = 50.66 \end{aligned}$$

- (b) Suppose the individual’s current wealth is ¥10,000. What is the maximum amount of money that she is willing to pay to avoid this risk?

Solution: The maximum amount of money she is now willing to pay, M , must solve

$$\begin{aligned} u(10000 - M) &= \frac{1}{2}u(10000 - 100) + \frac{1}{2}u(10000 - 0) \\ \sqrt{10000 - M} &= \frac{1}{2}\sqrt{9900} + \frac{1}{2}\sqrt{10000} \approx 99.75 \\ M &= 10000 - 9949.94 = 50.06 \end{aligned}$$

- (c) Now, suppose the individual’s current wealth is again ¥1000, but now the individual is facing the prospect of gaining ¥100 with probability $\frac{1}{2}$ and nothing with probability $\frac{1}{2}$. What is the maximum amount of money that she is willing to pay to take on this risk?

Solution: The maximum amount of money she is willing to pay, M , must solve

$$\begin{aligned}
 u(1000) &= \sqrt{1000} = \frac{1}{2}\sqrt{1100 - M} + \frac{1}{2}\sqrt{1000 - M} = \frac{1}{2}u(1100 - M) + \frac{1}{2}u(1000 - M) \\
 \sqrt{1100 - M} &= 2\sqrt{1000} - \sqrt{1000 - M} \\
 1100 - M &= 4000 - 2(2)\sqrt{1000}\sqrt{1000 - M} + (1000 - M) \\
 \sqrt{1000 - M} &= \frac{3900}{126.491} = 30.832 \implies M = 1000 - 30.832^2 = 49.39.
 \end{aligned}$$

That is, the decision maker will be willing to pay up to \$49.39 to take on this risk.

(d) Interpret these results.

Solution: Parts (a) and (b) shows that an individual's evaluation of uncertainty depends on the initial wealth the individual has. That the absolute value of the amount that needs to be paid in Parts (a) and (c) is different shows that an individual's evaluation of uncertainty involving loss may be different from that involving gain.