

Intermediate Microeconomics

Fall 2025 - M. Chen, M. Pak, and B. Xu

Problem Set 5: due in class, the week of Dec. 22

1. Consider an Edgeworth box economy where preferences are given by

$$u^1(x_1^1, x_2^1) = 2\sqrt{x_1^1 + x_2^1} \quad \text{and} \quad u^2(x_1^2, x_2^2) = 2\sqrt{x_1^2 + x_2^2},$$

- (a) Suppose the initial endowments are $e^1 = (4, 4)$ and $e^2 = (1, 1)$. Find all the Pareto optimal allocations.
- (b) Find the competitive equilibrium (also called Walrasian equilibrium).

2. Consider an Edgeworth box economy where preferences are given by

$$u^1(x_1^1, x_2^1) = x_1^1 + \ln x_2^1 \quad \text{and} \quad u^2(x_1^2, x_2^2) = x_1^2 x_2^2,$$

and the initial endowments are

$$e^1 = (1, 3) \quad \text{and} \quad e^2 = (3, 1).$$

Find all the competitive (Walrasian) equilibrium. You may assume that the solution is interior.

3. Consider a pure exchange economy with 2 consumers and 2 goods, where the consumers' preferences and endowments are given by

$$\begin{aligned} u^1(x_1^1, x_2^1) &= \min\{x_1^1, x_2^1\} & e^1 &= (6, 5) \\ u^2(x_1^2, x_2^2) &= \min\{x_1^2, x_2^2\} & e^2 &= (6, 5). \end{aligned}$$

- (a) Using a carefully labeled Edgeworth box diagram, graph the PO Set.
- (b) Using a carefully labeled Edgeworth box diagram, graph the contract curve, which is the subset of Pareto set where every consumer is doing as well as her endowment bundle.
- (c) Can allocation $(\hat{x}_1, \hat{x}_2) = ((8, 7), (4, 3))$ be supported as a Walrasian equilibrium with transfers? If so, find the supporting prices and the corresponding transfers. If not, explain.

4. Consider an exchange economy with:

$$u^1(x_1^1, x_2^1) = \min \left\{ x_1^1, \frac{x_2^1}{2} \right\} \quad e^1 = (3, 6)$$

$$u^2(x_1^2, x_2^2) = x_1^2 x_2^2 \quad e^2 = (2, 4).$$

- (a) Find the competitive equilibrium, using $p_2 = 1$ as normalization.
- (b) Is there any actual trade occurring in the equilibrium? Briefly explain what is driving this result.

5. Consider an Edgeworth box economy with two consumers, whose utility functions and endowments are

$$u^1(x_1^1, x_2^1) = (x_1^1)(x_2^1)^{\frac{1}{3}} \quad e^1 = (10, 0)$$

$$u^2(x_1^2, x_2^2) = (x_1^2)(x_2^2)^{\frac{1}{4}} \quad e^2 = (0, 10)$$

In the following, use the normalization $p_2 = 1$.

- (a) Find the competitive (Walrasian) equilibrium.
- (b) State the first fundamental theorem of welfare and verify that it holds in this economy.
- (c) Consider the allocation

$$\tilde{x} = (\tilde{x}^1, \tilde{x}^2) = ((2, 2.5), (8, 7.5)).$$

Can this allocation be supported as an equilibrium with transfers? If so, find the transfers and prices that will make this allocation an equilibrium with transfers.