

Intermediate Microeconomics

Fall 2025 - M. Chen, M. Pak, and B. Xu

Problem Set 1: due 4th meeting of the class.

1. Carefully sketch the indifference curves corresponding to the utility functions and the utility levels given below. Be sure to plot several (four or five) points that belong on each indifference curve.

(a)

$$u(x_1, x_2) = x_1^{\frac{1}{3}} x_2^{\frac{2}{3}}; \quad u_0 = 2 \text{ and } u_1 = 4.$$

(b)

$$u(x_1, x_2) = x_1 x_2^2; \quad u_0 = 8 \text{ and } u_1 = 64.$$

(c)

$$u(x_1, x_2) = \frac{1}{3} \ln x_1 + \frac{2}{3} \ln x_2; \quad u_0 = \ln 2 \text{ and } u_1 = \ln 4.$$

- (d) Explain how you would convince yourself that these indifference curves belong to the same individual and that the individual's preference ordering is convex and monotone.

2. Answer the following.

- (a) Suppose $f(\cdot)$ is an increasing function. Show that the utility functions $u(\cdot)$ and $\tilde{u}(\cdot) = f(u(\cdot))$ generate the same (Marshallian) demand.
- (b) Verify the above for the three utility functions in Question 1 by explicitly solving the corresponding utility maximization problems.
- (c) Notice that the marginal rates of substitution are the same for all three utility functions in part (b). More generally, show that a utility function $u(\cdot)$ and the transformed utility function $\tilde{u}(\cdot) = f(u(\cdot))$, where $f(\cdot)$ is an increasing function, have the same marginal rate of substitution. You may assume both $u(\cdot)$ and $f(\cdot)$ are differentiable.

3. Consider an individual with utility function

$$u(x_1, x_2) = \min \left\{ \frac{x_1}{a}, \frac{x_2}{b} \right\}.$$

- (a) How does the ratio of the optimal consumption of the two goods (specifically, $\frac{x_2(p_1, p_2, I)}{x_1(p_1, p_2, I)}$) change as prices change?

- (b) How does the ratio of optimal consumption of the two goods change as a and b change?
 - (c) Suppose the individual has income I and prices are p_1 and p_2 initially. Suppose the price of good 2 increases to $p'_2 > p_2$. Let ΔI be the (minimum) additional income that is needed to make the individual equally as well off as before the price change. Without doing any calculation, guess how ΔI changes as b changes. (You really should do this before part (d)).
 - (d) Verify your answer to part (c) with an explicit calculation.
4. Consider a firm that has two employees. Employee 1 has a preference ordering that can be represented by utility function $u_1(x_1, x_2) = \ln x_1 + \ln x_2$ while employee 2's preference ordering can be represented by $u_2(x_1, x_2) = x_1 + x_2$.
- (a) Sketch a few indifference curves (for example, ones going through bundles $(2, 2)$, $(4, 4)$, and $(6, 6)$, respectively) for each employee.
 - (b) Suppose the firm is located in city C , where the price levels are $(p_1, p_2) = (1, 1)$. It needs to send one of the two employees to its branch in city S . However, the price levels in city S are $(p'_1, p'_2) = (3, 1)$, so the firm may have to pay an additional salary to ensure that the employee is equally well-off in city S as she was in city C . If the firm wants to minimize the additional salary it needs to pay, which employee should it send? Explain using carefully labeled graphs.
 - (c) What if the price levels in city S were $p'' = (3, 2)$? Explain using carefully labeled graphs.
5. There are two goods in the economy: electricity (E) and the composite good (Y). Suppose the government subsidizes the first 20kWh of electricity consumption so that the price of electricity is ¥5 for first 20kWh and ¥10 thereafter, while the price of the composite good remains constant at $p_Y = ¥10$.
- (a) Placing the composite good on the vertical axis and electricity on the horizontal axis, carefully sketch the budget set for the consumer who has income ¥500.
 - (b) Suppose a consumer's utility function is given by $u(E, Y) = EY$. Find the consumer's utility maximizing bundle.