

Intermediate Microeconomics

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Midterm examination: suggested solutions

1. [20] Consider a two-period consumption model in which the individual's lifetime utility is $u(c_1, c_2) = \min\{c_1, \frac{c_2}{2}\}$, and the individual has income $I_1 > 0$ in period 1 and $I_2 \geq 0$ in period 2. Assume that the price levels are $p_1 = p_2 = 1$, and the individual can save or borrow freely at net interest rate $r > 0$.

- (a) [7] Suppose $I_2 = 0$. Find the optimal consumption in each period and the optimal savings.

Solution: Corner point equation and the lifetime budget constraint $(1+r)c_1 + c_2 = (1+r)I_1 + I_2$ yield

$$\begin{aligned} c_1 = \frac{c_2}{2} &\implies c_2 = 2c_1 \implies (1+r)c_1 + 2c_1 = (1+r)I_1 \\ \implies c_1^* &= \frac{(1+r)I_1}{3+r}, \quad c_2^* = \frac{2(1+r)I_1}{3+r}, \quad \text{and} \quad s^* = I_1 - c_1^* = \frac{2}{3+r}I_1. \end{aligned}$$

- (b) [7] Still assuming $I_2 = 0$, find the substitution effect, the income effect, and the total effect on the consumption in period 1 when the interest rate rises from $r = 0.1$ to $r' = 0.2$. Is the individual better off or worse off?

Solution: Since consumptions in period 1 and period 2 are perfect complements, the substitution effect is zero (it's easy to see that $c_1^* = c_1^h$). The new period 1 consumption is $c_1' = \frac{(1+r')I_1}{3+r'}$. Thus,

$$SE_1 = 0 \text{ and } IE_1 = TE_1 = c_1' - c_1^* = \frac{1.2I_1}{3.2} - \frac{1.1I_1}{3.1} = (0.375 - 0.355)I_1 = 0.02I_1.$$

Better off since $u^{\text{new}} - u^{\text{original}} = c_1' - c_1^* = 0.02I_1 > 0$. Note that since the individual was a saver before the interest rate rose and continues to be a saver afterwards, she is better off when the interest rate rises.

- (c) [6] Now suppose $I_2 > 0$. When will the individual become a borrower? Give an intuitive, verbal explanation of the condition you find.

Solution: Now, the lifetime income is $(1+r)I_1 + I_2$. Therefore, the condition to be the borrower, $c_1^* > I_1$, yields

$$\frac{(1+r)I_1 + I_2}{3+r} > I_1 \iff (1+r)I_1 + I_2 > (3+r)I_1 \iff I_2 > 2I_1.$$

The consumptions in period 2 and period 1 are perfect complements for the individual, and she needs to consume them at the ratio of 2:1, respectively

no matter what the relative price (interest rate) is. Since price levels are the same across periods, it means her spending ratio must also be 2:1. Thus, all that matters for her becoming a borrower or not is whether her period 2 income is greater than twice her period 1 income or not.

2. [20] Consider a leisure-consumption model in which an individual's utility over leisure and a consumption good is $u(L, Y) = 2L^{\frac{1}{2}} + Y$. Assume that the total amount of time available is $\bar{L}$, and let w be the wage (per unit of time) and p be the price of the consumption good.

- (a) [8] Find the condition under which the individual does not work.

Solution: If the solution is interior ($Y^* > 0$), we have

$$\begin{aligned} \text{MRS} = L^{-\frac{1}{2}} = \frac{w}{p} &\implies L^* = \left(\frac{p}{w}\right)^2 \implies w\left(\frac{p}{w}\right)^2 + pY^* = w\bar{L} \\ &\implies Y^* = \frac{w\bar{L} - \frac{p^2}{w}}{p} > 0 \iff w\bar{L} > \frac{p^2}{w}. \end{aligned}$$

Therefore, the individual will not work ($Y^* = 0$) if $w\bar{L} \leq \frac{p^2}{w} \iff w \leq \frac{p}{\bar{L}^{\frac{1}{2}}}$.

- (b) [4] Give an intuitive, verbal explanation of why the individual does not work in part (a) even though the individual derives positive utility from consumption.

Solution: Even though the individual derives positive utility from consumption, the individual is choosing to forego consumption in favor of leisure because the wage rate (price of leisure) is too low relative to the price of the consumption good. That is, working is not worthwhile because the amount of utility she would lose from leisure is not made up by the utility gain she would have from the consumption she could purchase with her earnings.

- (c) [8] To help the poor and stimulate consumption, the government gives a basic income B to the individual. (Basic income is a lump-sum cash given to the individual without any condition or requirement.) Suppose the government finds that the individual's consumption increases by $\frac{B}{p}$ but the individual still does not work. Explain whether the government should conclude that basic income can stimulate consumption but cannot encourage the individual to work.

Solution: To induce her to work, we need:

$$Y^* = \frac{w\bar{L} + B - \frac{p^2}{w}}{p} > \frac{B}{p} \iff w > \frac{p}{\bar{L}^{\frac{1}{2}}},$$

which is the same condition as before. Thus, in this example, a lump-sum cash payment cannot induce the individual to work. However, this is a consequence of the quasilinear utility we have assumed (MRS does not depend on Y) and need not hold in general (one may wish to verify it with a graphical example).

3. [20] A strictly risk averse individual has initial wealth W RMB and a lottery ticket. The lottery ticket pays 100 RMB with probability a and nothing with probability $1 - a$. Suppose an insurance company offers an “insurance” at a price of p RMB per unit. Each unit of insurance gives the individual 1 RMB if she does not win the lottery (i.e., gets 0 RMB in the lottery) and nothing if she wins (i.e., gets 100 RMB in the lottery).

- (a) [5] What is the individual’s expected utility from buying x units of insurance? Determine the fair price for the insurance.

Solution:

$$\begin{aligned} U(x) &= au(W - px + 100) + (1 - a)u(W - px + x) \\ &= au(W + 100 - px) + (1 - a)u(W + (1 - p)x). \end{aligned}$$

Next, $E[\pi] = px - (a(0) + (1 - a)x) = 0$ implies that the fair price is $p = 1 - a$.

- (b) [5] Find the optimal amount of insurance to buy at the fair price.

Solution:

$$\begin{aligned} U'(x) &= au'(W + 100 - px)(-p) + (1 - a)u'(W + (1 - p)x)(1 - p) = 0 \\ (1 - a)(1 - p)u'(W + (1 - p)x) &= apu'(W + 100 - px) \\ (1 - a)(a)u'(W + ax) &= a(1 - a)u'(W + 100 - (1 - a)x) \text{ since } p = 1 - a \\ W + ax &= W + 100 - (1 - a)x \implies x^* = 100. \end{aligned}$$

- (c) [5] Suppose the price of insurance is greater than the fair price. Show whether the individual will under insure, fully insure, or over insure.

Solution If $p > (1 - a)$, then $ap > (1 - p)(1 - a)$. Thus, using FOC from part (b) and the fact that $u'' < 0$ yield

$$\begin{aligned} u'(W + (1 - p)x) &= \frac{ap}{(1 - a)(1 - p)} u'(W + 100 - px) \\ u'(W + (1 - p)x) &> u'(W + 100 - px) \\ W + (1 - p)x &< W + 100 - px \implies x^* < 100 \text{ (under insure).} \end{aligned}$$

- (d) [5] Suppose the individual’s utility over money is $u(w) = \ln w$, and $a = \frac{1}{2}$ and $p = \frac{3}{4}$. Find the optimal amount of insurance to buy.

Solution: We have $\frac{ap}{(1 - a)(1 - p)} = \frac{\frac{3}{8}}{\frac{1}{4}} = 3$. Therefore, the FOC in part (c) becomes:

$$\begin{aligned} u'(W + (1 - \frac{3}{4})x) &= 3u'(W + 100 - \frac{3}{4}x) \\ \frac{1}{W + \frac{1}{4}x} &= \frac{3}{W + 100 - \frac{3}{4}x} \\ W + 100 - \frac{3}{4}x &= 3W + \frac{3}{4}x \\ 100 - 2W &= \frac{3}{2}x \implies x^* = \begin{cases} \frac{2(100 - 2W)}{3} & \text{if } W < 50 \\ 0 & \text{if } W \geq 50. \end{cases} \end{aligned}$$

4. [20] A firm's production function is $f(L, K) = L^{\frac{1}{9}} K^{\frac{2}{9}}$. Assume we are in the long-run time frame, and let w, r, p be the price of labor, capital, and the output good, respectively.

- (a) [10] Find the firm's optimal input choices and the cost function.

Solution: Using MRTS = price ratio and substituting the result into the output constraint yield,

$$\begin{aligned} \frac{\frac{1}{9} L^{-\frac{8}{9}} K^{\frac{2}{9}}}{\frac{2}{9} L^{\frac{1}{9}} K^{-\frac{7}{9}}} &= \frac{K}{2L} = \frac{w}{r} \implies K = \left(\frac{2w}{r}\right) L \implies f(L, K) = L^{\frac{1}{9}} \left(\frac{2w}{r}\right)^{\frac{2}{9}} L^{\frac{2}{9}} = L^{\frac{1}{3}} \left(\frac{2w}{r}\right)^{\frac{2}{9}} = q. \\ \implies L^* &= \left(\left(\frac{r}{2w}\right)^{\frac{2}{9}} q\right)^3 = \left(\frac{r}{2w}\right)^{\frac{2}{3}} q^3 \quad \text{and} \quad K^* = \frac{2w}{r} \left(\frac{r}{2w}\right)^{\frac{2}{3}} q^3 = \left(\frac{2w}{r}\right)^{\frac{1}{3}} q^3 \\ \implies c(q) &= w \left(\frac{r}{2w}\right)^{\frac{2}{3}} q^3 + r \left(\frac{2w}{r}\right)^{\frac{1}{3}} q^3 = \left(\left(\frac{1}{2}\right)^{\frac{2}{3}} + 2^{\frac{1}{3}}\right) w^{\frac{1}{3}} r^{\frac{2}{3}} q^3. \end{aligned}$$

- (b) [5] Find the firm's profit maximizing output level.

Solution:

$$p = MC \iff p = 3 \left(\left(\frac{1}{2}\right)^{\frac{2}{3}} + 2^{\frac{1}{3}} \right) w^{\frac{1}{3}} r^{\frac{2}{3}} q^2 \implies q^* = \frac{p^{\frac{1}{2}}}{3^{\frac{1}{2}} \left(\left(\frac{1}{2}\right)^{\frac{2}{3}} + 2^{\frac{1}{3}} \right)^{\frac{1}{2}} w^{\frac{1}{6}} r^{\frac{1}{3}}}.$$

- (c) [5] Suppose the government decides to collect t RMB in tax from the firm for each unit the firm sells. What happens to the firm's supply curve? What happens to the firm's supply curve if, instead of taxing the output good, the government decides to collect t RMB for each unit of labor the firm hires?

Solution: Firm now receives $p - t$ for each unit. Therefore, profit maximization condition is now

$$p - t = MC \iff p = MC + t.$$

Therefore, the firm's supply curve, which is the MC curve (note that minimum AC = 0), shifts up by t .

If instead the firm has to pay $w + t$ for each unit of labor, its MC becomes

$$MC \iff p = 3 \left(\left(\frac{1}{2}\right)^{\frac{2}{3}} + 2^{\frac{1}{3}} \right) (w + t)^{\frac{1}{3}} r^{\frac{2}{3}} q^2.$$

Therefore, the supply curve rotates up and becomes steeper.

5. [20] Consider a competitive market where the demand is given by $q^D = 100 - p$ and the (domestic) supply is given by $q^S = p$.

- (a) [5] Find the equilibrium and the aggregate surplus.

Solution: $q^D = q^S$ yields

$$100 - p = p \implies p^* = 50 \text{ and } q^* = 50 \implies AS = \frac{1}{2}(100)(50) = 2500.$$

- (b) [8] Suppose the market opens to international trade, and foreign producers enter the market. Suppose the foreign supply is given by $q^{FS} = 3p$ (note that foreign supply curve is upward sloping). What is the resulting change in the aggregate surplus (this is called the gains from the trade).

Solution: Now, the aggregate supply is $q^{AS} = q^{DS} + q^{FS} = 4p$. Thus,

$$100 - p = 4p \implies p^{**} = 20 \text{ and } q^{**} = 80.$$

However, note that domestic producer sells only $q^{DS} = p = 20$.

$$CS^n = \frac{1}{2}(100 - 20)(80) = 3200 \text{ and domestic } PS^n = \frac{1}{2}(20 - 0)(20) = 200$$

$$AS^n = 3400 \implies \text{gains from the trade} = AS^n - AS = 3400 - 2500 = 900.$$

- (c) [7] Suppose the government decides to impose an ad valorem tax to foreign producers (but not the domestic producers). That is, for each unit foreign producers sell in this market, they have to give tp to the government, where $t = \frac{1}{3}$ is the tax rate and p is the price the buyers pay. What is the tax revenue and the incidence of tax? Is it the case that this tax on import only hurts the foreign producers?

Solution: Now, $q^{FS} = 3(1 - \frac{1}{3})p = 2p$, which means $q^{AS} = 3p$. Thus,

$$100 - p^D = 3p^D \implies p^D = 25 \text{ and } q^T = 75 \implies q^{DS} = 25 \text{ and } q^{FS} = 50.$$

$$GR = tp \times \text{import} = \frac{1}{3}(25)(50) = 416.67.$$

$$\text{incidence on the consumers} = \frac{25 - 20}{\frac{25}{3}} = \frac{75 - 60}{25} = 0.6$$

$$\text{incidence on the foreign producers} = \frac{20 - \frac{2}{3}(25)}{\frac{25}{3}} = \frac{60 - 50}{25} = 0.4.$$

Therefore, 40% of the tax burden falls on the domestic consumers.