Intermediate Microeconomics

Fall 2024 - M. Chen, M. Pak, and B. Xu

Problem Set 4: suggested solutions

1. [Optional - not to be graded] Consider a market where there are two consumers with inverse demand functions $p(q_1) = 10 - q_1$ and $p(q_2) = 5 - q_2$.

   (a) Suppose there is a single firm with inverse supply function $p(q) = \frac{1}{2}q$. Find the competitive equilibrium.

   Solution: Adding the two individual demands yield

$$\begin{aligned} q_1 &= 10 - p \\ q_2 &= 5 - p \\ \Rightarrow Q^D &= \begin{cases} 10 - p & \text{if } 5 \le p \le 10 \\ 15 - 2p & \text{if } p \le 5 \end{cases} \end{aligned}$$

   Since there is only one firm, the market supply function is $Q^s = 2p$. The equilibrium condition $Q^D = Q^S$ has no solution when $5 \le p \le 10$. So, using the portion of Q^D below $p \le 5$ yields:

$$\begin{aligned} Q^D &= Q^S \\ 15 - 2p &= 2p \\ \Rightarrow \quad p^* &= 3.75 \\ Q^* &= 2(3.75) = 7.5. \end{aligned}$$

   (b) Find the elasticity of demand and supply at the equilibrium.

   Solution: The two elasticities are

$$\begin{aligned} \varepsilon_D &= \frac{\partial Q^D}{\partial P}\left(\frac{P}{Q^D}\right) \\ &= (-2)\left(\frac{3.75}{7.5}\right) = -1, \quad \text{and} \\ \eta_D &= \frac{\partial Q^S}{\partial P}\left(\frac{P}{Q^S}\right) \\ &= (2)\left(\frac{3.75}{7.5}\right) = 1 \end{aligned}$$

   (c) Suppose instead that there are three firms with the identical inverse supply function given in part (a). Find the competitive equilibrium.

Solution: Now, $Q^s = 6p$. Again, $Q^D = Q^S$ has no has no solution when $5 \leq p \leq 10$. Using the portion of Q^D below $p \leq 5$ yields:

$$\begin{aligned} Q^D &= Q^S \\ 15 - 2p &= 6p \\ \Rightarrow \quad p^* &= 1.875 \\ Q^* &= 6(1.875) = 11.25. \end{aligned}$$

2. Consider a market where the supply and the demand are given by

$$Q^S(P) = 100P \quad \text{and} \quad Q^D(P) = 2000 - 100P.$$

(a) Find the equilibrium price, quantity, consumer surplus, producer surplus, and the aggregate surplus.

Solution:

$$\begin{aligned} & Q^S(P) = 100P = 2000 - 100P = Q^D(P) \\ \Rightarrow \quad & P^* = 10 \quad \text{and} \quad Q^* = 100(10) = 1,000. \end{aligned}$$

The surpluses are

$$\begin{aligned} CS &= \frac{1}{2}(20 - 10)(1000) = 5,000 \\ PS &= \frac{1}{2}(10 - 0)(1000) = 5,000 \\ AS &= 10,000. \end{aligned}$$

(b) Suppose the government wants to raise revenue by imposing tax of ¥4 per unit. What is the price producers get, the price consumers pay, the equilibrium quantity, the tax revenue, and the dead weight loss?

Solution: We now set

$$\begin{aligned} Q^S(P) = 100(P^S) &= 2000 - 100(P^S + t) = Q^D(P) \\ 200P^S &= 2000 - 100t \end{aligned}$$

To obtain

$$\begin{aligned} P^S &= 10 - \frac{t}{2} = 8 \\ P^D &= 8 + 4 = 12 \\ Q^T &= 800 \\ GR &= 4(800) = 3,200 \\ DWL &= (0.5)(t)(Q^* - Q^T) = (0.5)(4)(1000 - 800) = 400. \end{aligned}$$

(c) Suppose the government is thinking about imposing an ad valorem tax instead of per unit tax. What does the tax rate has to be to keep the price consumers pay the same as in the per unit tax case?

Solution: Setting

$$Q^S(P) = 100(P^S) = 2000 - 100(P^S + tP^S) = Q^D(P)$$

yields

$$200P^S + 100tP^S = 2000 \quad \Rightarrow \quad P^S = \frac{2000}{200 + 100t}$$

We want

$$\begin{aligned} (1+t)P^S = 12 \quad &\Rightarrow \quad (1+t)\frac{2000}{200+50} = 12 \\ &\Rightarrow \quad 2000 + 2000t = 2400 + 1200t \\ &\Rightarrow \quad t = 50\% \end{aligned}$$

We have

$$\begin{aligned} P^S &= \frac{2000}{200 + 100(.5)} = 8 \\ P^T &= (1 + 0.5)P^S = 12 \end{aligned}$$

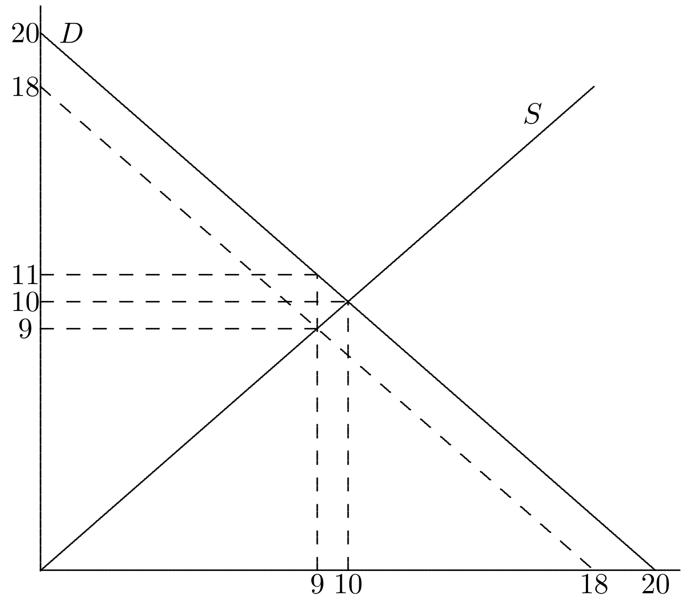
(d) Which tax scheme is better for the economy? Why?

Solution: Note that even in valorem case, $Q^T = 100P^S = 800$. Since this means that CS, PS, GR, and DWL are all the same under both tax schemes. So they are equivalent.

3. Consider a market where the supply is given by $Q^S = P$ and the demand is given by $Q^D = 20 - P$.

   (a) Suppose the government wants to raise 18¥ by imposing per unit tax on this market. What tax rate will raise the required revenue and also minimize the dead weight loss?

Solution:



Letting $P^D = P^S + t$, we obtain

$$\begin{aligned} Q^D &= 20 - (P^S + t) = P^S = Q^S \\ \Rightarrow \quad P^S &= \frac{20-t}{2} \\ Q^S(t) &= \frac{20-t}{2} \\ \text{Setting} \quad 18 = GR &= tQ^s(t) = t\left(\frac{20-t}{2}\right), \\ \text{yields} \quad t^2 - 20t + 36 &= 0 \\ (t-2)(t-18) &= 0 \\ \Rightarrow \quad t &= 2 \text{ or } 18. \end{aligned}$$

Since

$$Q^s(2) = 9 \; > \; 1 = Q^s(18),$$

$t = 2$ will minimize the dead weight loss.

(b) What is the resulting equilibrium and the dead weight loss? What is the incidence of taxation?

Solution: Quantity traded is $Q^S = 9$, $P^S = 9$, and $P^D = 11$. The dead weight loss is $DWL = (0.5)(2)(10 - 9) = 1$, and the incidence of tax is $\frac{11-10}{2} = 0.5$ for the consumer and $\frac{10-9}{2} = 0.5$ for the producer.

(c) Can you think of a method for raising 18¥ from this market that will (1) incur no dead weight loss and (2) be preferable to the per unit tax for both the producers and the consumers.

Solution: Since the tax burden is split equally, the government can simply demand lump-sum tax of 9¥ from the producers and 9¥ from the consumers. Since quantity traded remains the same as the competitive equilibrium, this will result in no dead weight loss. Hence, both the producers and the consumers will be better off under this scheme than the per unit tax.

4. Consider again the market where the supply and the demand are given by

$$Q^S(P) = 100P \quad \text{and} \quad Q^D(P) = 2000 - 100P.$$

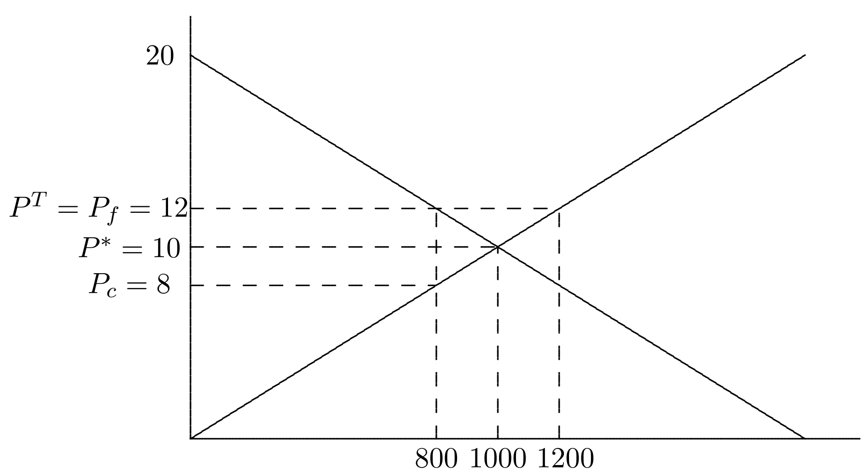
(a) Suppose the government imposes a price floor of $P_f = 12$. Assuming that the government buys any excess supply, find the consumer surplus, producer surplus, government revenue, and aggregate surplus. What is the deadweight loss (in comparison to the no intervention case)?

Solution: Since $P_f = 12 > 10 = P^*$, the price floor is binding. The quantity demanded and supplied at this price is given by

$$\begin{aligned} Q^S(P_f) &= 100P_f = 100(12) = 1200 \\ Q^D(P_f) &= 2000 - 100(P_f) = 2000 - 100(12) = 800 \\ \text{Excess supply} &= Q^S(P_f) - Q^D(P_f) = 1200 - 800 = 400. \end{aligned}$$

The relevant surpluses are

$$\begin{aligned} CS' &= \tfrac{1}{2}(20 - P_f) \times Q^D(P_f) = \tfrac{1}{2}(20 - 12)(800) = 3200 \\ PS' &= \tfrac{1}{2}(P_f - 0) \times Q^S(P_f) = \tfrac{1}{2}(12 - 0)(1200) = 7200 \\ GR' &= -P_f \times \text{excess supply} = 12(400) = -4800 \\ AS' &= CS' + PS' + GR' = 3200 + 7200 - 4800 = 5600 \\ DWL &= AS - AS' = 10000 - 5600 = 4400. \end{aligned}$$



(b) Suppose the government imposes a price ceiling $p_c = 8$. What is the change in consumer surplus from the no intervention case?

Solution: Since $P_c = 8 < 10 = P^*$, the price ceiling is binding. This means that there will be a shortage. That is, the quantity transacted will be whatever the producers are willing to supply at price P_c. That is $Q^T = Q^S(P_c) = 100(8) = 800$. The price that the consumers are willing to pay for Q^T is given by

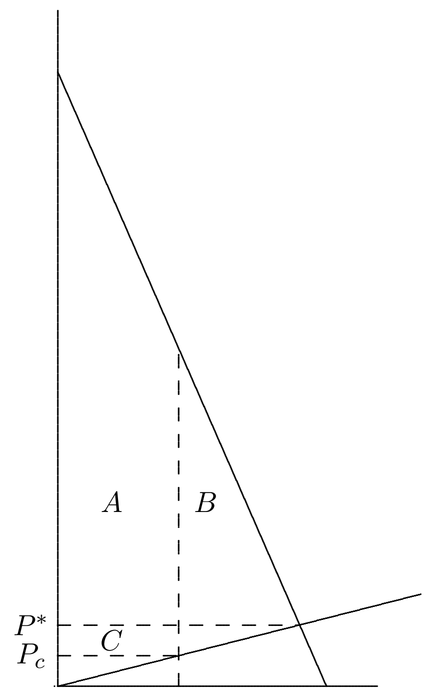
$$Q^T = Q^D(P^T) \iff 800 = 2000 - 100(P^T) \iff P^T = 12.$$

Thus,

$$\begin{aligned} CS' &= \tfrac{1}{2}(20 - P^T) \times Q^T + (P^T - P_c) \times Q^T \\ &= \tfrac{1}{2}(20 - 12)(800) + (12 - 8)(800) = 3200 + 3200 = 6400 \end{aligned}$$
$$\implies CS' - CS = 6400 - 5000 = 1400.$$

(c) In general (i.e., do not restrict attention to the supply and the demand functions given above), will consumers always benefit from price ceiling? (A carefully drawn graphical example will suffice).

Solution: When demand curve is relatively steep (not price sensitive) and supply curve is relatively flat (price sensitive), then price ceiling can actually reduce consumer surplus. In the picture below, consumer surplus without intervention is $CS = A + B$. When price ceiling P_c is imposed, the new consumer surplus is $CS' = A + C$. In the picture, the difference $CS' - CS = C - B$ is negative.



5. Consider a market where the domestic supply and the domestic demand are given by

$$Q^S(P) = 200P \quad \text{and} \quad Q^D(P) = 4000 - 200P.$$

(a) Suppose this is a closed economy. Find the equilibrium price and quantity. What are the consumer surplus, producer surplus, and aggregate surplus?

Solution: Setting $Q^s = Q^D$ yields

$$\begin{aligned} 200P &= 4000 - 200P \\ P^* &= 4000/400 = 10 \\ \Rightarrow \quad Q^* &= 200(10) = 2000. \end{aligned}$$

Inverse supply and demand functions are:

$$P = \frac{Q^S}{200} \quad \text{and} \quad P = 20 - \frac{Q^D}{200}.$$

$$\begin{aligned} \Rightarrow \quad CS^* &= 0.5(20-10)(2000) = 10,000 \\ PS^* &= 0.5(10-0)(2000) = 10,000 \\ AS^* &= CS^* + PS^* = 20,000. \end{aligned}$$

For the remainder of the question, assume that the economy is open.

(b) The world supply is perfectly elastic at price $P_w = 5$. Find the equilibrium price, quantity traded, and total import. What are the consumer surplus, producer surplus, aggregate surplus, and gains from the trade?

Solution: Since the world supply is perfectly elastic at $P_w < P^*$, the equilibrium price will fall to $P_w = 5$.

$$\begin{aligned} \text{Quantity traded} &= Q^D(P_w) = 4000 - 200(5) = 3,000 \\ \text{Domestic supply} &= Q^S(P_w) = 200(5) = 1,000 \\ \text{Import} &= Q^D(P_w) - Q^S(P_w) = 3000 - 1000 = 2,000 \\ CS^o &= 0.5(20-5)(3000) = 22,500 \\ PS^o &= 0.5(5-0)(1000) = 2,500 \\ AS^o &= 25,000 \\ \text{Gains from the trade} &= AS^o - AS^* = 5,000. \end{aligned}$$

(c) Suppose the government wants to help the domestic producers by limiting the imports to 1,200 units. Find the domestic equilibrium price, quantity traded, and supply. Assuming that the price of the imported goods will rise to the same level as the domestically produced goods, find the consumer surplus, producer surplus and aggregate surplus. What is the dead weight loss compared to free trade?

Solution: Import $= Q^D - Q^S$. So, if imports are restricted to 1,200 units, domestic price will rise until

$$\begin{aligned}
Q^D(P) - Q^S(P) &= 4000 - 200P - 200P = 1,200 \\
P^q &= 2800/400 = 7 \\
\text{Quantity traded} &= Q^D(P^q) = 4000 - 200(7) = 2,600 \\
\text{Domestic supply} &= Q^S(P^q) = 200(7) = 1,400 \\
\text{Import} &= 1,200 \\
CS^q &= 0.5(20-7)(2600) = 16,900 \\
PS^q &= 0.5(7-0)(1400) = 4,900 \\
AS^q &= 21,800 \\
DWL^q &= 3,200.
\end{aligned}$$

(d) Suppose the government wants to reduce the imports to 1,200 units by using tariffs rather than direct quota. How should the government set the tariff to achieve this? Compare the dead weight loss under tariff and quota.

Solution: From Part (c), we see that the effective price has to be $p^q = 7$ to reduce the imports to 1200. Therefore, tariff needs to be $t = 2$.

$$\begin{aligned}
CS^t &= 0.5(20-7)(2600) = 16,900 \\
PS^t &= 0.5(7-0)(1400) = 4,900 \\
GR^t &= (\text{tariff})(\text{import}) = 2(1200) = 2,400 \\
AS^t &= 21800 + 2400 = 24,200 \\
DWL^t &= 800.
\end{aligned}$$

Since the effect of the import tariff is identical to the quota, except that government gets to collects tax revenue, it results in smaller dead weight loss.

(e) Can you think of reasons why a government may want to use quota rather than tariff?

Solution: Any reasonable answer will do. Here's one possibility. In practice, it's difficult to measure how effective import tariff is in reducing foreign competition. In contrast, quota system gives the government direct control over the level imports. Therefore, if the government wants to be certain about the degree to which it is protecting the domestic producers, it may wish to use quota despite the higher social cost.