Intermediate Microeconomics

Exercises: Competitive Market, Exchange economy, and Monopoly

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3 Competitive Market and Government Policies

1. Consider a market where each existing firm and potential entrants have the identical long-run cost function $c(q) = 10q^3 - 5q^2 + 20q$. Market demand function is $Q^D = 39000 - 2000p$.

   (a) In the long-run, what is the equilibrium market price, market output, number of firms and output per firm?

   Solution: In the long-run equilibrium, firms produce at output level where $p = MC = AC$ (recall min AC occurs where MC and AC curves intersect). So,

$$\begin{aligned} MC &= AC \\ \Rightarrow \quad 30q^2 - 10q + 20 &= 10q^2 - 5q + 20 \\ 20q^2 - 5q &= 0 \\ q^* &= 0.25, \quad \text{and} \\ p^* &= MC \\ &= 30(0.25)^2 - 10(0.25) + 20 \\ &= 19.375 \end{aligned}$$

   So, the equilibrium price is $p^* = 19.375$ and each firm in the market will produce $q^* = 0.25$ units. Market output is

$$Q^* = Q^D(p^*) = 39000 - 2000(19.375) = 250,$$

   and the number of firms in the market is $Q^*/q^* = 250/.25 = 1000$.

   (b) Suppose a technological innovation reduces each firm's cost function to $c(q) = 10q^3 - 6q^2 + 20q$. Compare the new long-run equilibrium with that found in part (a).

   Solution: Now $MC = AC$ yields

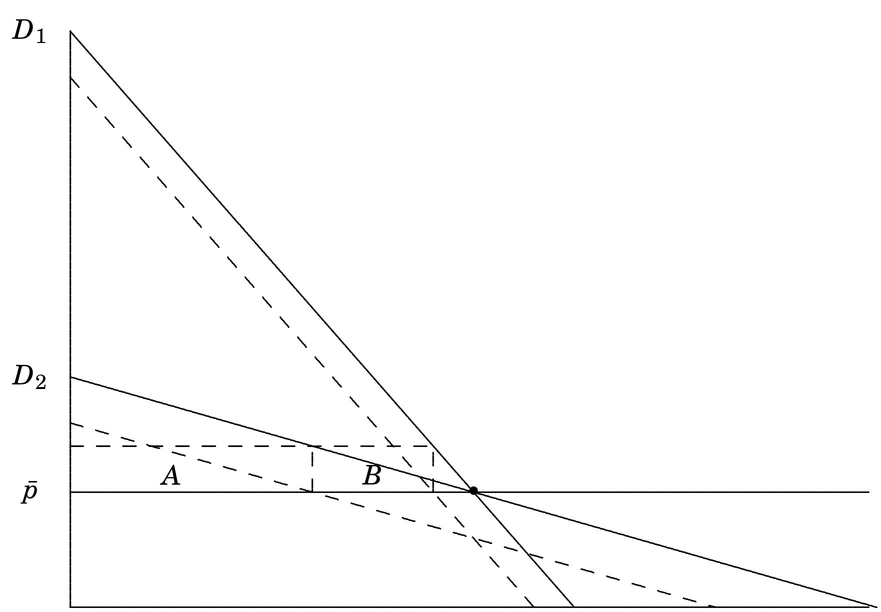
$$\begin{aligned} 30q^2 - 12q + 20 &= 10q^2 - 6q + 20 \\ 20q^2 - 6q &= 0 \\ \Rightarrow \quad q^* &= 0.3, \\ p^* &= 30(0.30)^2 - 12(0.30) + 20 \; = \; 19.1, \\ Q^* &= 39000 - 2000(19.1) \; = \; 800, \quad \text{and} \\ N &= 800/0.3 = 2667.67 \end{aligned}$$

That is, when the firms' costs were reduced, the equilibrium price fell to $p^* = 19.1$ and market output increased to 800 units. Both the number of firms in the market and the production level of each firm in the market increased to 2,667.67 and 300, respectively.

2. Consider a government that wants to raise revenue by implementing per unit tax of t on a commodity. The government has a choice of taxing one of two markets. Both markets have the usual downward-sloping, linear demand curves. However, the demand curve in market 1 is steeper than the demand curve in market 2 (that is, the demand in market 1 is relatively less elastic than market 2). To keep the situation simple, assume that the two markets have exactly the same supply curve that is perfectly elastic at price $\bar{p}$. Assume further that the equilibrium quantity prior to taxation is also the same in both markets.

   (a) What are some possible reasons for taxing market 1 rather than market 2?

   Solution: As seen in the graph, more inelastic demand implies less quantity reduction by the consumers. So, the tax revenue will be higher. In the graph below, revenue from taxing market 1 is $A + B$ while revenue from market 2 is only A.



   (b) What are some possible reasons for taxing market 2 rather than market 1?

   Solution: More tax revenue necessarily in this example means greater burden on the consumer.

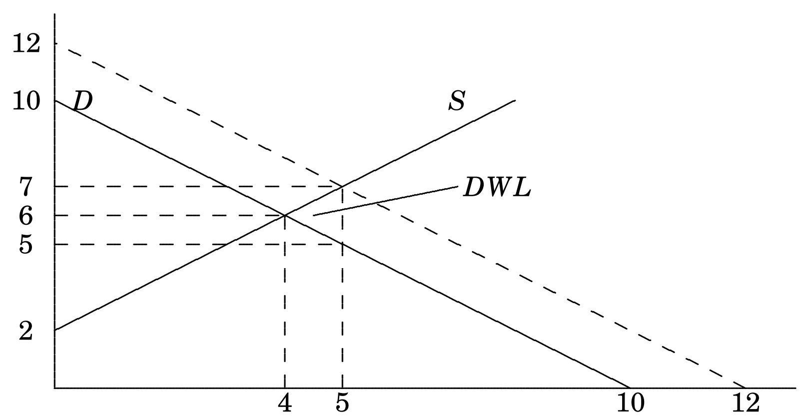
(c) Use your answers above to explain whether government should tax cigarettes.

Solution: Any serious answer will do. Here are some possibilities. Since demand for cigarette is inelastic, taxing cigarette may be an effective way to generate revenue. However, cigarette demand is inelastic because of its addictive effect. Therefore, taxing this market may be viewed as an exploitation of the unfortunate addicts. Of course, one may wish to incorporate the health dimension and the associated cost of cigarette smoking into the discussion.

3. Consider a market where the supply is given by $Q^S = P - 2$ and the demand is given by $Q^D = 10 - P$.

(a) Find the competitive equilibrium. What is the consumer, the producer, and the aggregate surplus?

Solution:



Competitive equilibrium is found by:

$$
\begin{aligned}
Q^D = 10 - P &= P - 2 = Q^S \\
\Rightarrow \quad P^* &= 6 \\
Q^* &= 4. \\
CS^* &= (0.5)(10-6)(4) = 8 \\
PS^* &= (0.5)(6-2)(4) = 8.
\end{aligned}
$$

(b) Suppose the government wants to encourage production by instituting a subsidy of 2¥ per unit. What is the impact of the subsidy on the quantity traded, the prices, the consumer surplus and the producer surplus?

Solution: Subsidy works like negative tax. That is $P^D = P^S - s$, where s

denotes the subsidy. Thus, we have

$$\begin{aligned} Q^D = 10-(P^S-2) &= P^s-2 = Q^S \\ \Rightarrow \quad P^S &= 7 \\ P^D &= 5 \\ \hat{Q} &= 5. \\ \hat{CS} &= (0.5)(10-5)(5) = 12.5 \\ \hat{PS} &= (0.5)(7-2)(5) = 12.5 \end{aligned}$$

(c) Suppose the subsidy the government pays will have to be raised by levying lump-sum tax on the consumers. What is the impact of the subsidy on the consumer's welfare? What is the impact on the welfare if the tax burden is shared equally by the consumers and the producers?

Solution: Government revenue is $-(s)(\hat{Q}) = -2(5) = -10$. If the entire amount comes from the consumers, their total surplus is now

$$\widetilde{CS} = 12.5-10 = 2.5 < 8 = CS^*.$$

If the burden is shared equally, then

$$\begin{aligned} \widetilde{CS} &= 12.5-5 = 7.5 < 8 = CS^* \\ \widetilde{PS} &= 12.5-5 = 7.5 < 8 = PS^*. \end{aligned}$$

Note that the deadweight loss from the subsidy is -1. One way or the other, the burden must be born by the consumers and/or the producers.

4. Consider again the market where the domestic supply and the domestic demand are given by

$$Q^S(P) = 200P \quad \text{and} \quad Q^D(P) = 4000-200P.$$

(a) Suppose, now the world price is $P_w = 12$, which is above the closed economy equilibrium price, P^*. As usual, assume that the world demand is perfectly elastic at $P_w = 12$ and that when indifferent between selling in the domestic market and the foreign market, producers sell in the domestic market first. Find the equilibrium price, quantity traded domestically, and total export. What are the consumer surplus, the producer surplus, and the aggregate surplus. Who gains and/or loses from trading?

Solution: Since $P_w > P^*$ domestic producers will sell to the domestic consumers at price P_w and sell the remainder in the foreign market

$$\begin{aligned} \text{Quantity traded domestically} &= Q^D(P_w) = 4000-200(12) = 1,600 \\ \text{Domestic supply} &= Q^S(P_w) = 200(12) = 2,400 \\ \text{Export} &= Q^S(P_w)-Q^D(P_w) = 2400-1600 = 800 \\ CS^o &= 0.5(20-12)(1600) = 6,400 \\ PS^o &= 0.5(12-0)(2400) = 14,400 \\ AS^o &= 20,800 \\ \text{Gains from the trade} &= AS^o - AS^* = 20800-20000 = 800. \end{aligned}$$

When the economy is opened, the domestic price is raised to the world price. As a result, domestic consumers loses $CS^* - CS^o = 10000 - 6400 = 3,600$ while producers gain $PS^o - PS^* = 14400 - 1000 = 4,400$. Since producer's gains are greater than the consumer's loss, there is a net gain in the aggregate surplus of $-3600 + 4400 = 800$.

(b) Suppose the government wants to encourage export by giving a subsidy of $s = 1$ per unit of export. Find the equilibrium price, quantity traded, and total import. What are the consumer surplus, the producer surplus, and the dead weight loss? Who wins and loses from the subsidy?

Solution: Subsidy raises the price that the producer gets from exporting to $P^s = P_w + s = 13$. Thus, domestic producers try to sell to the domestic consumers at price P^s and sell the remainder in the foreign market. However, whether the domestic consumer will buy the good at P^s depends on whether they have the option to buy the import at P_w.

If they cannot buy imports, then they will be forced to accept P^s so that

$$\begin{aligned}
\text{Quantity traded domestically} &= Q^D(P^s) = 4000 - 200(13) = 1,400 \\
\text{Domestic supply} &= Q^S(P^s) = 200(13) = 2,600 \\
\text{Export} &= Q^S(P^s) - Q^D(P^s) = 2600 - 1400 = 1,200 \\
CS^s &= 0.5(20-13)(1400) = 4,900 \\
PS^s &= 0.5(13-0)(2600) = 16,900 \\
GR^s &= -s(\text{export}) = -(1)(1200) = -1,200 \\
AS^s &= 20,600 \\
DWL^s &= AS^s - AS^o = 20800 - 20600 = 200.
\end{aligned}$$

As seen above, the subsidy increases the producer's surplus at the further expense of the consumers.

But, if the consumers can buy imports, then they will buy imports at price P_w rather than the domestically produced goods at P^s. Therefore, domestic producer will sell all of its output in the foreign market.

$$\begin{aligned}
\text{Quantity traded domestically} &= Q^D(P_w) = 4000 - 200(12) = 1,600 \\
\text{Domestic supply} &= Q^S(P^s) = 200(13) = 2,600 \\
\text{Export} &= Q^S(P^s) - 0 = 2600 = 2,600 \\
CS^s &= 0.5(20-12)(1600) = 6,400 \\
PS^s &= 0.5(13-0)(2600) = 16,900 \\
GR^s &= -s(\text{export}) = -(1)(2600) = -2,600 \\
AS^s &= 6400 + 16900 - 2600 = 20,700 \\
DWL^s &= AS^s - AS^o = 20800 - 20700 = 100.
\end{aligned}$$

5. Consider a market where the domestic supply and the domestic demand are given by

$$Q^S(P) = 100P \quad \text{and} \quad Q^D(P) = 2000 - 100P.$$

Assume that the economy is open and that the world supply and the world demand is perfectly elastic at price $P_w = 5$.

(a) Find the equilibrium price and the quantity traded. Is the country a net exporter or importer? What are the consumer surplus, producer surplus, aggregate surplus, and gains from the trade?

Solution: The closed economy equilibrium is given by

$$\begin{aligned} & Q^D(P) = 2000 - 100P \;=\; 100P = Q^S(P) \\ \Rightarrow\quad & P^c = \tfrac{2000}{200} = 10 \text{ and } Q^c = 1000 \\ \Rightarrow\quad & AS = CS + PS = \tfrac{1}{2}(20-10)(1000) + \tfrac{1}{2}(10-0)(1000) = 10{,}000. \end{aligned}$$

The open economy equilibrium is given by

$$\begin{aligned} P^* = 5 \quad\Rightarrow\quad & Q^S = 500 \text{ and } Q^D = 1500 \quad\Rightarrow\quad \text{net importer} \\ & CS = \tfrac{1}{2}(20-5)(1500) = 11{,}250 \\ & PS = \tfrac{1}{2}(5-0)(500) = 1250 \\ & AS = 12{,}500 \text{ and gains from trade} = 2{,}500. \end{aligned}$$

(b) Suppose the government wants to reduce the imports to 500 units by using tariffs. How should the government set the tariff to achieve this? Find the deadweight loss from the tariff.

Solution: We need

$$\begin{aligned} & Q^D(5+t) - Q^S(5+t) = 2000 - 100(5+t) - 100(5+t) = 500 \\ \Rightarrow\quad & 2000 - 1000 - 200t = 500 \\ & \Rightarrow\quad t = \tfrac{500}{200} = 2.5 \\ & AS = \tfrac{1}{2}(20-7.5)(1250) + \tfrac{1}{2}(7.5)(750) + 2.5(500) \\ & \quad = 7812.5 + 2812.5 + 1250 = 11{,}875 \\ & DWL = 12{,}500 - 11{,}875 = 625. \end{aligned}$$

(c) Suppose there is a technological change in the domestic firms so that the domestic supply is now given by $Q^S(P) = 400P$ while everything else remains the same. Find the new equilibrium price and the quantity traded. Is the country a net exporter or importer? What are the consumer surplus, producer surplus, aggregate surplus, and gains from the trade?

Solution: The closed economy equilibrium is now given by

$$\begin{aligned} & Q^D(P) = 2000 - 100P \;=\; 400P = Q^S(P) \\ \Rightarrow\quad & P^c = \tfrac{2000}{500} = 4 \text{ and } Q^c = 1600 \\ \Rightarrow\quad & AS = CS + PS = \tfrac{1}{2}(20-4)(1600) + \tfrac{1}{2}(4-0)(1600) = 12{,}800 + 3{,}200 \\ & = 16{,}000. \end{aligned}$$

The open economy equilibrium is given by

$$\begin{aligned} P^* = 5 \quad\Rightarrow\quad & Q^S = 2000 \text{ and } Q^D = 1500 \quad\Rightarrow\quad \text{net exporter} \\ & CS = \tfrac{1}{2}(20-5)(1500) = 11{,}250 \\ & PS = \tfrac{1}{2}(5-0)(2000) = 5000 \\ & AS = 16{,}250 \text{ and gains from trade} = 250. \end{aligned}$$

4 Exchange Economy

1. Consider again the Edgeworth box economy in Problem Set 5, Question 2. Can the allocation $(x^1, x^2) = ((1,1),(3,3))$ be supported as a price equilibrium with transfers? If yes, find the supporting prices and transfers. If no, explain.

   Solution:
   Yes. Allocation (x^1, x^2) is feasible and Parento Optimal.

$$\begin{aligned} MRS^1 &= x_2^1 = 1 \\ MRS^2 &= \frac{x_2^2}{x_1^2} = \frac{3}{3} = 1 \\ \Rightarrow MRS^1 &= MRS^2 \end{aligned}$$

   So, by the Second Welfare Theorem, we can support this allocation as an equilibrium with transfers. The supporting prices are given by the MRS of the consumers at this allocation. That is, $\frac{p_1}{p_2} = 1$. With our normalization, we get $p_1 = 1, p_2 = 1$. So, for transfers,

$$\begin{aligned} T_1 &= (p_1, p_2)\cdot(x_1^1, x_2^1) - (p_1, p_2)\cdot(e_1^1, e_2^1) \\ &= (1,1)\cdot(1,1) - (1,1)\cdot(1,3) = -2 \\ T_2 &= -T_1 = 2 \end{aligned}$$

2. Consider an Edgeworth box economy with two consumers, whose utility functions and endowments are

$$\begin{aligned} u^1(x_1^1, x_2^1) &= (x_1^1)(x_2^1)^{\frac{1}{3}} \qquad e^1 = (5,5) \\ u^2(x_1^2, x_2^2) &= (x_1^2)(x_2^2)^{\frac{1}{4}} \qquad e^2 = (5,5) \end{aligned}$$

   In the following, use the normalization $p_2 = 1$.

   (a) Find the competitive equilibrium price.

   Solution: We first transform the utilities into a standard Cobb-Douglas ones:

$$\begin{aligned} u^1(x_1^1, x_2^1) &= (x_1^1)(x_2^1)^{\frac{1}{3}} \sim (x_1^1)^{\frac{3}{4}}(x_1^1)^{\frac{1}{4}} \\ u^2(x_1^2, x_2^2) &= (x_1^2)(x_2^2)^{\frac{1}{4}} \sim (x_1^2)^{\frac{4}{5}}(x_2^2)^{\frac{1}{5}}. \end{aligned}$$

   The incomes of the two consumers are

$$\begin{aligned} I_1 &= 5p_1 + 5 \\ I_2 &= 5p_1 + 5 \end{aligned}$$

The Marshallian demand functions are

$$\begin{aligned}
x_1^1(p_1,p_2) &= \frac{3}{4}\left(\frac{5p_1+5}{p_1}\right) = \frac{15p_1+15}{4p_1} \\
x_2^1(p_1,p_2) &= \frac{1}{4}\left(\frac{5p_1+5}{p_2}\right) = \frac{5p_1+5}{4} \\
x_1^2(p_1,p_2) &= \frac{4}{5}\left(\frac{5p_1+5}{p_1}\right) = \frac{20p_1+20}{5p_1} \\
x_2^2(p_1,p_2) &= \frac{1}{5}\left(\frac{5p_1+5}{p_2}\right) = \frac{5p_1+5}{5}.
\end{aligned}$$

We'll seek to clear the second market:

$$\begin{aligned}
x_2^1(p_1,p_2)+x_2^2(p_1,p_2) &= \frac{25p_1+25}{20}+\frac{20p_1+20}{20} = \frac{45p_1+45}{20} = 10 \\
45p_1 &= 200-45 \implies \hat{p}_1 = \frac{155}{45} = 3.44.
\end{aligned}$$

So, the equilibrium price is $\hat{p}=(3.44,1)$.

(b) State the first fundamental theorem of welfare and verify that it holds in this economy.

Solution: The first fundamental theorem of welfare states that under a mild set of conditions (preferences are monotone), every competitive equilibrium is Pareto optimal. In the current example, the competitive equilibrium allocation is

$$\begin{aligned}
\hat{x}_1^1 &= \frac{3}{4}\left(\frac{5(3.44)+5}{3.44}\right) = \frac{3}{4}\left(\frac{22.2}{3.44}\right) = 4.84 \\
\hat{x}_2^1 &= \frac{1}{4}\left(\frac{5(3.44)+5}{1}\right) = \frac{1}{4}\left(\frac{22.2}{1}\right) = 5.55 \\
\hat{x}_1^2 &= \frac{4}{5}\left(\frac{5(3.44)+5}{3.44}\right) = \frac{4}{5}\left(\frac{22.2}{3.44}\right) = 5.16 \\
\hat{x}_2^2 &= \frac{1}{5}\left(\frac{5(3.44)+5}{1}\right) = \frac{1}{5}\left(\frac{22.2}{1}\right) = 4.44.
\end{aligned}$$

Note that $\hat{x}_1^2+\hat{x}_2^2=9.99$, which is slightly under 10 due to rounding errors. At the equilibrium allocation, we have

$$\begin{aligned}
MRS^1 &= \frac{(x_2^1)^{\frac{1}{3}}}{\frac{1}{3}(x_1^1)(x_2^1)^{-\frac{2}{3}}} = \frac{3(x_2^1)}{(x_1^1)} = \frac{(3)(5.55)}{4.84} = 3.44 \\
MRS^2 &= \frac{(x_2^2)^{\frac{1}{4}}}{\frac{1}{4}(x_1^2)(x_2^2)^{-\frac{4}{5}}} = \frac{4(x_2^2)}{(x_1^2)} = \frac{4(4.44)}{5.16} = 3.44.
\end{aligned}$$

So, it is indeed Pareto optimal.

(c) Consider the allocation $\tilde{x}=(\tilde{x}^1,\tilde{x}^2)=\big((2,3),(8,7)\big)$. Show whether this allocation can supported as an equilibrium with transfers.

Solution:

$$
\begin{aligned}
MRS^1 &= \frac{(x_2^1)^{\frac{1}{3}}}{\frac{1}{3}(x_1^1)(x_2^1)^{-\frac{2}{3}}} = \frac{3(x_2^1)}{(x_1^1)} = \frac{(3)3}{2} = \frac{9}{2} = 4.5 \\
MRS^2 &= \frac{(x_2^2)^{\frac{1}{4}}}{\frac{1}{4}(x_1^2)(x_2^2)^{-\frac{4}{5}}} = \frac{4(x_2^2)}{(x_1^2)} = \frac{4(7)}{8} = \frac{28}{8} = 3.5.
\end{aligned}
$$

Because the indifference curves of the two consumers are not tangent at allocation $\tilde{x}$, it cannot be supported as an equilibrium with transfers.

(d) State the second fundamental theorem of welfare, and briefly discuss whether the result in part (b) conform with or violate this theorem.

Solution: The second fundamental theorem of welfare states that under a fairly reasonable conditions on the preferences (monotonicity and convexity), every Pareto optimal allocation can be supported as an equilibrium with transfers. Note that it only guarantees that Pareto optimal allocations are supportable. Thus, since allocation $\tilde{x}$ is not Pareto optimal, the fact that it is not supportable as an equilibrium with transfer does not violate the second welfare theorem.

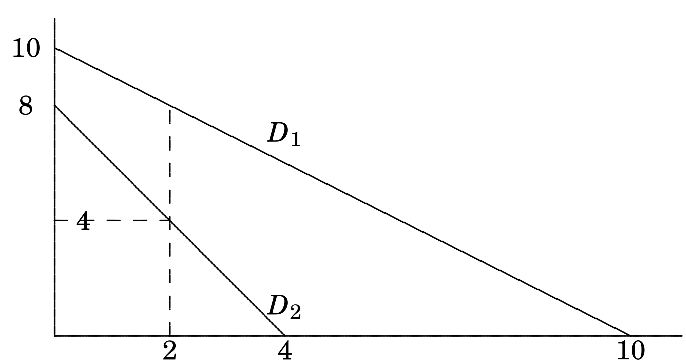
5 Monopoly

1. A monopolist with zero cost, that is $c(q) = 0$, faces two consumers whose demand functions are given below.

$$
\begin{aligned}
Q_1 &= 10 - P \\
Q_2 &= 4 - \frac{1}{2}P
\end{aligned}
$$

(a) Suppose the monopolist cannot engage in any price discrimination. Find the firm's optimal pricing strategy.

Solution:



The inverse demand functions of the consumers are:

$$
\begin{aligned}
P_1 &= 10 - Q \\
P_2 &= 8 - 2Q.
\end{aligned}
$$

The aggregate demand and the inverse aggregate demand is give by

$$
\begin{aligned}
Q &= \begin{cases} 10-P & \text{if } 8 \le P \le 10 \\ 14-\frac{3}{2}P & \text{if } 0 \le P \le 8 \end{cases} \\
P &= \begin{cases} 10-Q & \text{if } 0 \le Q \le 2 \\ \frac{28}{3}-\frac{2}{3}Q & \text{if } 2 \le Q \le 14 \end{cases}
\end{aligned}
$$

The corresponding marginal cost curve is

$$
MR = \begin{cases} 10-2Q & \text{if } 0 \le Q \le 2 \\ \frac{28}{3}-\frac{4}{3}Q & \text{if } 2 \le Q \le 14 \end{cases}
$$

The optimal pricing strategy is found by setting $MR = MC$, which yields

$$
\begin{aligned}
\frac{28}{3}-\frac{4}{3}Q &= 0 \\
28-4Q &= 0 \\
Q^m &= 7 \\
P^m &= \frac{28}{3}-\frac{2}{3}(7) = \frac{14}{3}.
\end{aligned}
$$

(b) Now, assume that price discrimination is possible. Find the monopolist's optimal first degree price-discrimination strategy.

Solution: Generally, first degree price-discrimination strategy involves the firm selling each unit of the good to the consumer who values it most at exactly the consumer's valuation, thereby extracting all the surpluses from the consumers. Here, the marginal cost is constant, so the monopolist can simply offer the quantity - total price pair that will extract all the surpluses to each consumer.

The total surplus available to the consumers are

$$
\begin{aligned}
CS_1 &= (0.5)(10)(10) = 50, \quad \text{and} \\
CS_2 &= (0.5)(8)(4) = 16.
\end{aligned}
$$

So, the firm should offer $(Q,TP) = (10,50)$ to consumer 1 and $(Q,TP) = (4,16)$ to consumer 2.

(c) Find the monopolist's optimal second degree price-discrimination strategy.

Solution: To find the quantity to be offered to consumer, we set

$$
\begin{aligned}
P_1 &= 2P_2 \\
10-Q &= 2(8-2Q) \\
10-Q &= 16-4Q \\
\Rightarrow \quad Q &= 2
\end{aligned}
$$

The total surplus available to consumer 2 at $Q = 2$ is $0.5(8-4)(2)+(4)(2) = 12$. So the monopolist should offer $(Q_2, TP_2) = (2,12)$ in the menu. If consumer 1 takes the option $(2,12)$ her total surplus will be $0.5(10-8)(2) + 0.5(8-4)(2) = 2+4 = 6$. So, the monopolist should offer $(Q_1, TP_1) = (10, 50-6) = (10,44)$ as the other option.

2. Consider a monopolist with constant marginal cost $MC(Q) = 1$ facing two consumers. Consumer 1 has demand function $Q_1^D = 5 - P$ while consumer 2 has $Q_2^D = 10 - 2P$.

   (a) Suppose the monopolist acts competitively and uses $P = MC$ pricing strategy. Find the resulting equilibrium price and quantity. Find the consumer, producer, and aggregate surplus.

   Solution: Aggregate demand is given by

$$\begin{aligned} Q^D(P) &= Q_1^D(P) + Q_2^D(p) = (5-P) + (10-2P) = 15 - 3P \\ \Rightarrow \quad P &= 5 - \frac{1}{3}Q^D \end{aligned}$$

   Setting $P = MC$ yields

$$\begin{aligned} 5 - \frac{Q}{3} &= 1 \\ \Rightarrow \quad Q^* &= 4(3) = 12 \\ P^* &= 1 \\ \Rightarrow \quad CS^* &= 0.5(5-1)(12) = 24 \\ PS^* &= 0 \\ AS^* &= 24. \end{aligned}$$

   (b) Suppose the monopolist uses $MR = MC$ pricing strategy. Find the resulting equilibrium price and quantity. Find the effect on consumer, producer, and aggregate surplus.

   Solution: Revenue is $P(Q)Q = \left(5 - \frac{1}{3}Q\right)Q$. So, $MR = MC$ yieds

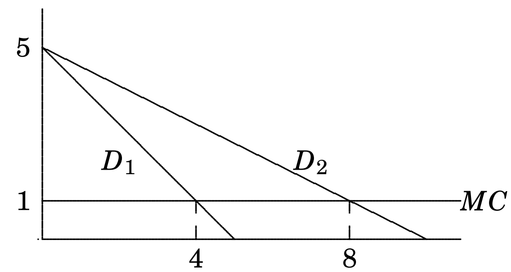
$$\begin{aligned} 5 - \frac{2Q}{3} &= 1 \\ \Rightarrow \quad Q^m &= \frac{4(3)}{2} = 6 \\ P^m &= P(Q^m) = 5 - \frac{1(6)}{3} = 3 \\ \Rightarrow \quad CS^* &= 0.5(5-3)(6) = 6 \\ PS^m &= (3-1)(6) = 12 \\ AS^m &= 18. \end{aligned}$$

   As the result of monopoly pricing, producer surplus increased at the expense of the consumers and overall social inefficiency ($DWL = AS^* - AS^m = 24 - 18 = 6$).

   (c) Suppose the monopolist uses 1st degree price discrimination. Find the resulting equilibrium prices and quantities. Find the consumer, producer, and aggregate surplus.

Solution: The consumer's demand and inverse demand functions are given by:

$$Q_1^D = 5 - P \iff P_1 = 5 - Q_1$$
$$Q_2^D = 10 - 2P \iff P_2 = 5 - \frac{1}{2}Q_2.$$



At $p = MC = 1$, the monopolist should sell 4 units to consumer 1 and 8 units to consumer 2. Consumer 1's total available surplus is $CS_1^* = 0.5(5-1)(4) + 1(4) = 12$ and $CS_2^* = 0.5(5-1)(8) + 1(8) = 24$. Therefore, by offering 4 units at the total price of ¥12 to consumer 1 and 8 units at the total price of ¥24 to consumer 2, it can capture all of the available consumer surplus. As a result,

$$\begin{aligned} CS^{1^\circ} &= 0 \\ PS^{1^\circ} &= \text{REV} - \text{COST} = (12+24) - (4+8) = 24 \\ AS^{1^\circ} &= 24. \end{aligned}$$

Note that we need to subtract the rectangular areas from the monopolist's revenue since that area represents cost of production.

3. Consider a market with one firm. The firm's cost function is $c(q) = 2q$, and the market demand is $Q^D = 1000 - \frac{1}{2}P$.

   (a) Suppose the monopolist does not exercise any market power and behaves like a competitive firm. Find the equilibrium price, the quantity produced and the firm's profit.

   Solution: If the monopolist behaves like a competitive firm, then the firm sets $P = MC$. Since

   $$Q^D = 1000 - \tfrac{1}{2}P \iff P = 2000 - 2Q^D,$$

   we have

   $$\begin{aligned} P = MC \iff 2000 - 2Q = 2 \implies Q^* &= \frac{1998}{2} = 999 \\ P^* &= 2 \\ \pi^* &= P^*Q^* - c(Q^*) = 2(999) - 2(999) = 0. \end{aligned}$$

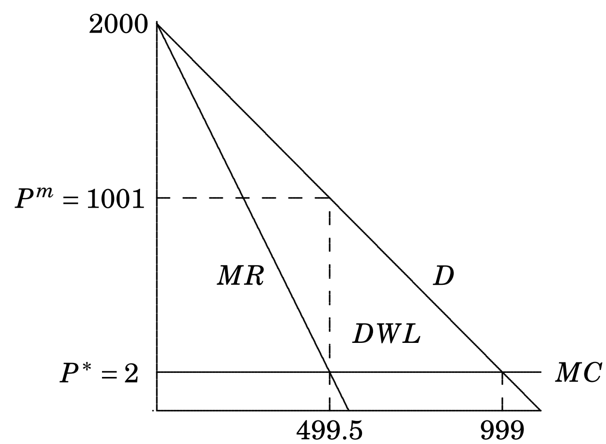
(b) Suppose the monopolist exercises market power but does not price discriminate (that is, the firm uses $MR = MC$ pricing strategy). Find the price the firm charges, the quantity produced, and the firm's profit. Also find the dead weight loss (in comparison to the competitive market outcome).

Solution: The firm solves

$$\max_{Q} (2000 - 2Q)Q - 2Q$$

Solving the first order condition yields

$$\begin{aligned} MR &= 2000 - 4Q \;=\; 2 = MC \\ Q^m &= \frac{1998}{4} = 499.5 \\ P^m &= 2000 - 2(Q^m) = 2000 - 2(499.5) = 1001 \\ \pi^m &= P^m Q^m - c(Q^m) = (1001)(499.5) - 2(499.5) = 499,000.5 \\ DWL &= \tfrac{1}{2}(1001 - 2)(999 - 499.5) = (0.5)(999)(499.5) = 249,500.25. \end{aligned}$$



(c) Now suppose price discrimination is possible. Find the monopolist's optimal first degree price-discrimination strategy. What is the dead weight loss (in comparison to the competitive market outcome)?

Solution: In the first-degree price discrimination, the monopolists extracts all the possible surpluses from the consumer, which is the area under the demand curve, upto the point where the demand curve intersects the marginal cost curve. Thus, the monopolists offers $Q^{1^\circ} = 999$ and the total price charged is

$$TP^{1^\circ} = \tfrac{1}{2}(2000 - 2)(999) + 2(999) = 998001 + 1998 = 999,999.$$

The dead weight loss is zero.

4. Consider a monopolist with zero cost, $C(Q) = 0$, facing two consumers. Consumer 1's demand is given by $P = 8 - \frac{1}{2}Q$ while consumer 2's is $P = 8 - Q$. Suppose the monopolist can engage in second-degree price discrimination.

   (a) What menu of quantity-price pairs should it offer to its customers? What's the firm's profit?

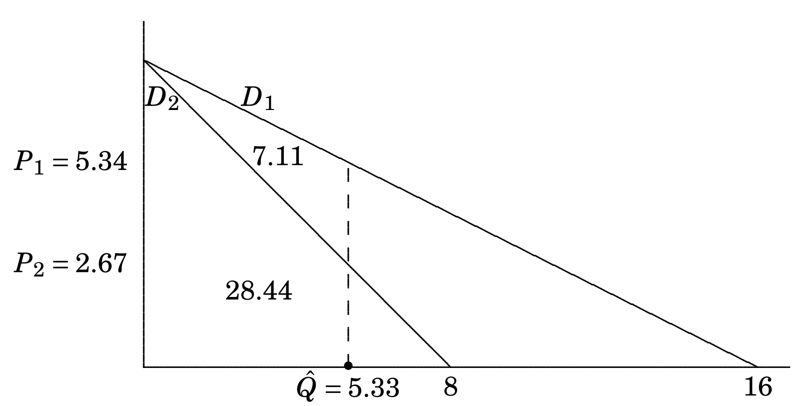
   Solution: To distinguish per unit price and total purchase price, let's use P for the first and TP for the latter. If the monopolist offers the product at its marginal cost, ($P = MC = 0$), then the two consumers' surpluses are $CS_1 = \frac{1}{2}(8)(16) = 64$ and $CS_2 = \frac{1}{2}(8)(8) = 32$. So, the second-degree price discrimination strategy can be found in the following way. Start at quantity-price pair that will capture all of consumer 2's surplus, $(Q, TP) = (8, 32)$. Then reduce the quantity (and the corresponding purchase price) until the marginal gain in revenue from consumer 1 is equal to the loss in revenue from consumer 2.

$$\begin{aligned} 8 - Q &= \left(8 - \frac{1}{2}Q\right) - (8 - Q) \\ 16 - 2Q &= 8 - \frac{1}{2}Q \\ 32 - 4Q &= 16 - Q \\ \hat{Q} &= \frac{16}{3} = 5.33. \end{aligned}$$

   The two consumers' willingness to pay and their total available surplus at $\hat{Q} = 5.33$ is

$$\begin{aligned} P_1(\hat{Q}) &= 8 - \frac{1}{2}(5.33) = 5.34 \\ P_2(\hat{Q}) &= 8 - 5.33 = 2.67 \\ CS_1(\hat{Q}) &= 0.5(8 - 5.34)(5.33) + 5.34(5.33) = 35.55 \\ CS_2(\hat{Q}) &= 0.5(8 - 2.67)(5.33) + 2.67(5.33) = 28.44. \end{aligned}$$

   If the firm offers 5.33 units at total price of 28.44, and consumer 1 chooses this quantity pair, her total surplus will be $35.55 - 28.44 = 7.11$. So, to induce consumer 1 to buy 16 units, the firm cannot charge more than $64 - 7.11 = 56.89$. So, the optimal menu is to offer $(Q, TP) = (5.33, 28.44)$ and $(Q, TP) = (16, 56.89)$.



(b) Suppose that consumer 1's demand is instead given by $P = 4 - \frac{1}{4}Q$. What menu of quantity-price pairs should it offer to its customers? What's the firm's profit?

Solution: At $P = MC = 0$, consumer 1's total available surplus is $CS_1 = 0.5(4)(16) = 32$ and consumer 2's total available surplus is $CS_2 = 0.5(8)(8) = 32$. The firm can extract all the surpluses from the consumers by offering the menu $(Q, TP) = (8, 32)$ and $(Q, TP) = (16, 32)$. To see that consumer 1 will not accept $(8, 32)$ note that consumer 1's willingness to pay at $Q = 8$ is $P(8) = 4 - \frac{1}{4}(8) = 2$, and her total available surplus from buying 8 units is only $0.5(4-2)(8) + 2(8) = 8 + 16 = 24 < 32$. The firm's profit is $32 + 32 = 64$.

5. Consider a monopolist with cost function $c(q) = 20q$ facing two consumers. The consumer's demand functions are given by

$$Q_1 = 100 - p$$
$$Q_2 = 50 - \frac{p}{2}.$$

(a) Suppose the monopolist does not price discriminate. Find the monopolist's optimal pricing strategy and the resulting profit.

Solution: The inverse demand functions are given by

$$p = 100 - Q_1$$
$$p = 100 - 2Q_2$$

Since they have the same intercept, we can add the two demand functions to obtain the aggregate demand function:

$$Q = 150 - \frac{3}{2}p \quad \Longleftrightarrow \quad p = 100 - \frac{2}{3}Q$$

Setting $100 - \frac{4}{3}Q = MR = MC = 20$ yields

$$Q^* = 60$$
$$p^* = 100 - \frac{2}{3}(60) = 100 - 40 = 60$$
$$\pi^* = (60 - 20)60 = 2,400$$

(b) Suppose the monopolist can engage in first degree price discrimination. Find the monopolist's optimal pricing strategy and the resulting profit.

Solution: To find the consumer surpluses, firs we find the quantities each consumer demands when $p = MC$:

$$\bar{Q}_1 = 100 - 20 = 80$$
$$\bar{Q}_2 = 50 - \frac{20}{2} = 40$$

Total price to charge to the consumers are:

$$TP_1 = (0.5)(100-20)(80) + 20(80) = 3200 + 1600 = 4800$$
$$TP_2 = (0.5)(100-20)(40) + 20(4) = 1600 + 800 = 2400$$

So, the monopolist will offer a menu with two quantity-price pair: offer (80 units at total price 4,800) to consumer 1 and (40 unites at total price 2,400) to consumer 2. The resulting profit is ¥4,800.

(c) Now, suppose the monopolist can produce goods at zero cost. That is $c(q) = 0$. Find the monopolist's optimal second degree price discrimination strategy.

Solution: (Need more detail)

The monopolist is going to offer $\hat{Q}_2$ to consumer 2, where $\hat{Q}_2$ satisfies:

$$100 - \hat{Q}_2 = 2(100 - 2\hat{Q}_2)$$
$$\hat{Q}_2 = \frac{100}{3} = 33.33$$

(d) Suppose the two consumers' demand function had been:

$$Q_1 = 100 - p$$
$$Q_2 = 100 - 4p.$$

Find the monopolist's optimal second degree price discrimination strategy and the resulting profit when the firm's cost function is still $c(q) = 0$.

Solution: (Need more detail)

The inverse demand functions are

$$p = 100 - Q_1$$
$$p = 25 - \frac{Q_2}{4}.$$

The monopolist is going to offer $\hat{Q}_2$ to consumer 2, where $\hat{Q}_2$ satisfies:

$$100 - \hat{Q}_2 = 2(25 - \frac{\hat{Q}_2}{4})$$
$$\hat{Q}_2 = (50)(2) = 100.$$

So $\hat{Q}_2 = 0$. Thus, firm will not sell to consumer 2.

6. A monopolist with cost function $c(q) = q$ faces two consumers whose demand functions are given below.

$$Q_1 = 100 - P$$
$$Q_2 = 50 - \tfrac{1}{2}P$$

(a) Suppose the monopolist cannot engage in any price discrimination. Find the firm's optimal pricing strategy. Calculate the firm's Lerner index.

Solution: The aggregate demand is given by

$$Q = 150 - \tfrac{3}{2}P \quad \Rightarrow \quad P = 100 - \tfrac{2}{3}Q$$

If the firm employs $MR = MC$ pricing, we have

$$\begin{aligned} MR &= 100 - \tfrac{4}{3}Q \ = \ 1 = MC \\ Q &= \tfrac{99(3)}{4} = 74.25 \\ P &= 100 - \tfrac{2}{3}\left(\tfrac{99(3)}{4}\right) = 100 - \tfrac{99}{2} = 49.5 = 50.5 \\ LI &= \frac{P - MC}{P} = \frac{50.5 - 1}{50.5} = \frac{49.5}{50.5} = \frac{99}{101} \approx 0.98 \end{aligned}$$

(b) What is the deadweight loss associated with this pricing strategy, if any?
Solution: If the firm acted competitively, it will use $P = MC$ pricing.

$$\begin{aligned} P = MC \quad &\Rightarrow \quad Q = 150 - \tfrac{3}{2} = 148.5 \\ AS = CS + PS &= \tfrac{1}{2}(100-1)(148.5) + 0 = 7,350.75 \end{aligned}$$

Under $MR = MC$ pricing,

$$\begin{aligned} AS' = CS' + PS' &= \tfrac{1}{2}(100 - 50.5)(74.25) + (50.5 - 1)(74.25) \\ &= 1837\tfrac{11}{16} + (49.5)(74.25) = 1837.6875 + 3675.375 = 5,513.0625. \\ DWL = AS - AS' &= 7350.75 - 5513.0625 = 1,837.6875. \end{aligned}$$

(c) Now, assume that price discrimination is possible. Find the monopolist's optimal first-degree price-discrimination strategy and the associated deadweight loss.
Solution: First-degree price discrimination extracts all the available surplus from each consumer. Since inverse demand functions are $P_1 = 100 - Q_1$ and $P_2 = 100 - 2Q_2$, we have

$$\begin{aligned} P_1 = MC &\implies 100 - Q_1 = 1 \implies Q_1 = 99 \\ CS_1(99) &= \tfrac{1}{2}(100-1)(99) + 1(99) = 4900.5 + 99 = 4,999.5 \\ P_2 = MC &\implies 100 - 2Q_2 = 1 \implies Q_2 = \tfrac{99}{2} = 49.5 \\ CS_2(49.5) &= \tfrac{1}{2}(100-1)(49.5) + 1(49.5) = 2450.25 + 49.5 = 2,499.75. \end{aligned}$$

Thus, the monopolist offers $(Q_1, TP_1) = (99, 4,999.5)$ to consumer 1 and $(Q_2, TP_2) = (49.5, 2,495.75)$ to consumer 2. The aggregate surplus is

$$AS^{1^\circ} = CS^{1^\circ} + PS^{1^\circ} = 0 + \pi_1 + \pi_2 = 4900.5 + 2450.25 = 7,350.75,$$

and $DWL = AS - AS^{1^\circ} = 7350.75 - 7350.75 = 0$.

(d) Find the monopolist's optimal third-degree price-discrimination strategy.
Solution: Now the monopolists just uses $MR = MC$ for each consumer.

$$\begin{aligned} MR_1 = 100 - 2Q_1 \ = \ 1 = MC \quad &\Rightarrow \quad Q_1 = \tfrac{99}{2} = 49.5 \\ P_1 = 100 - 49.5 = 50.5 & \\ MR_2 = 100 - 4Q_2 \ = 1 \ = MC \quad &\Rightarrow \quad Q_1 = \tfrac{99}{4} = 24.75 \\ P_2 = 100 - 2(24.75) = 100 - 49.5 = 50.5. & \end{aligned}$$

Note that the price charged to each consumer is the same in this example because the elasticities happen to be the same by coincidence:

$$\varepsilon_1 = \frac{dQ_1}{dP_1}\left(\frac{P_1}{Q_1}\right) = -1\left(\frac{50.5}{49.5}\right) \quad \text{and} \quad \varepsilon_2 = \frac{dQ_2}{dP_2}\left(\frac{P_2}{Q_2}\right) = -\frac{1}{2}\left(\frac{50.5}{24.75}\right) = -\frac{50.5}{49.5}.$$

7. A monopolist with **upward sloping** marginal cost curve $MC = \frac{2}{3}Q$ faces two consumers whose demand functions are

$$Q_1^D = 3 - P \quad \text{and} \quad Q_2^D = 4 - \frac{2}{3}P.$$

Suppose the monopolist can engage in 1st degree price discrimination.

(a) To whom should the monopolist sell the first good to and at what price?

Solution: The inverse demands functions of the two consumers are

$$P_1 = 3 - Q \quad \text{and} \quad P_2 = 6 - \frac{3}{2}Q.$$

Recall that 1st degree price discrimination requires the monopolist to sell each good to the consumer who values it the most. Since

$$\max\left\{3-1\ ,\ 6-\frac{3}{2}(1)\right\} = \max\left\{2\ ,\ \frac{9}{2}\right\} = \frac{9}{2},$$

the first unit should be sold to consumer 2 at price $\frac{9}{2}$.

(b) To whom should the monopolist sell the second good to and at what price?

Solution: Since consumer 2 has already bought one unit of the good while consumer 1 has not, the price that each consumer is willing to pay to obtain this additional unit is given by $P_1(1)$ and $P_2(2)$. Since

$$\max\left\{3-1\ ,\ 6-\frac{3}{2}(2)\right\} = \max\{2\ ,\ 3\} = 3,$$

the second unit should also be sold to consumer 2 at price 3.

(c) To whom should the monopolist sell the third good to and at what price?

Solution: Consumer 2 has already bought two units of the good while consumer 1 has not. So, the price that each consumer is willing to pay to obtain this additional unit is given by $P_1(1)$ and $P_2(3)$. Since

$$\max\left\{3-1\ ,\ 6-\frac{3}{2}(3)\right\} = \max\left\{2\ ,\ \frac{3}{2}\right\} = 2,$$

the third unit should be sold to consumer 1 at price 2. Note that when MC is upward sloping, the customer order may matter.