Gender Ratio under China's Two-Child Policy*

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Abstract

China's one-child policy has often been criticized for exacerbating its gender imbalance. Although such criticism implies that the gender imbalance should improve significantly once the one-child policy is removed, experiences of other countries with similar gender imbalance and no mandated fertility limit suggest that this conclusion should not be accepted without closer examination. Consequently, this paper examines the effects of allowing parents to have two children on the gender ratio. Specifically, we build a model of parental decision-making, in which parents choose between letting nature decide the gender of their child and manipulating the birth process to increase the likelihood of obtaining a son, and identify the optimal behaviors in this framework. We investigate the equilibrium level of gender imbalance under both the one-child and the two-child policy settings and show through a series of examples that the gender imbalance need not improve under the two-child policy.

Key words: Gender imbalance, One-child policy, Two-child policy
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1 Introduction

Researchers have long cited China's one-child policy as an important contributor to its persistently high gender imbalance (Zeng et al. [19]; Banister [2]; Das Gupta [5]; Hesketh and Xing [16]).[1] Although these earlier works did not attempt to formally establish causality, the prevailing view has been that the fertility limit imposed by the policy, combined with a strong preference for sons, leads parents to manipulate the birth process to obtain a son.[2] More recent works by Ebenstein [9] and Li et al. [14] have sought to establish a direct link between the one-child policy and the gender imbalance. Li et al. in particular attribute about 54% of the increases in (male-to-female) gender ratios for the 2001-2005 birth cohorts to the one-child policy.

At first glance, these findings suggest that the gender imbalance should improve substantially as China moves toward a two-child policy, as recently announced by the Chinese policymakers.[3] However, such expectation deserves a closer examination since countries without a one-child policy, such as India, Vietnam, and South Korea, have also experienced similar gender imbalances (Guilmoto [11]; Park and Cho [15]; Chun and Das Gupta [4]). Notably, South Korea experienced a dramatic rise in its gender ratio at birth from 107.4 in 1983 to 116.5 in 1990 and an equally dramatic fall back to 107.7 by 2005, all during which the fertility rate remained below two (Chun and Das Gupta). Therefore, even though low fertility rates, in conjunction with son-preference, may be driving gender imbalance, it is not clear at the onset whether relaxing the fertility limit from one to two will make a significant difference.

Intuitively, if there are no constraints on the number of children that parents can have, then a son-preference would not necessarily lead to a high gender ratio. For example, if preference for sons takes the form of "desire for at least one son," and parents satisfy their desires by continuing to have a child until a son is born, then the gender ratio will be balanced rather than being skewed toward boys.[4] In contrast, a binding constraint on the number of children, whether imposed by a policy or by

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[1] According to the censuses conducted by China's National Bureau of Statistics, there were 113.8 male births to every 100 female births in 1990. The figure rose to 119.9 in 2000 and then declined slightly to 118.1 in 2010. Normal gender ratio at birth ranges from 105 to 107 (Hesketh and Xing).

[2] As noted by Das Gupta et al. [6] and Ding and Zhang [8], one important source of son-preference in China is its cultural norm, which dictates that a married couple's primary elder care duty is to the parents of the male spouse.

[3] In November 2013, the CPC Central Committee loosened the one-child policy by allowing couples to have two children if at least one of the couple is an only child.

[4] To simplify the argument, assume that the probability of producing a boy or a girl is equal. Let $\mathrm{E}[\#B]$ and $\mathrm{E}[\#G]$ be the expected number of boys and girls that are produced by a couple following this rule. Then, letting $\mathrm{Prob}(g_1...g_n)$ be the probability of having n children, with the gender of the k-th child being g_k, we have $\mathrm{E}[\#B] = 1$ and

$$\begin{aligned}
\mathrm{E}[\#G] &= (0)\mathrm{Prob}(B) + (1)\mathrm{Prob}(GB) + (2)\mathrm{Prob}(GGB) + (3)\mathrm{Prob}(GGGB) + (4)\mathrm{Prob}(GGGGB) + \cdots \\
&= 0\left(\tfrac{1}{2}\right) + 1\left(\tfrac{1}{4}\right) + 2\left(\tfrac{1}{8}\right) + 3\left(\tfrac{1}{16}\right) + 4\left(\tfrac{1}{32}\right) + \cdots \\
&= \tfrac{1}{4}\left(\sum_{n=0}^{\infty}\frac{1}{2^n}\right) + \tfrac{1}{8}\left(\sum_{n=0}^{\infty}\frac{1}{2^n}\right) + \tfrac{1}{16}\left(\sum_{n=0}^{\infty}\frac{1}{2^n}\right) + \cdots \;=\; \tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{8} + \cdots \;=\; 1.
\end{aligned}$$

child-rearing costs, creates an incentive to manipulate the birth process to obtain a son before the constraint is reached.

When the constraint is one, this incentive induces a simple behavior. Those with strong enough desire for a son manipulate the birth process and the rest do not. However, the incentive generates more complex behaviors when the constraint is two. Some parents may manipulate both of their births, while others never manipulate. Some may let nature decide the gender of their first child but manipulate their second birth if they did not obtain a son. Still others may manipulate first, and then let nature decide their second child. The proportions of parents switching from manipulating, or not manipulating, under the one-child policy to these and possibly other behaviors ultimately determine whether the gender imbalance will improve. Therefore, this paper builds a model to explore how parents' gender-manipulation decisions change with the policy, and we show through a series of examples that gender ratio need not improve under a two-child policy.

Theoretical work on gender preference has been limited. Two notable studies investigated the effects of gender preference on fertility and investments in children (Davies and Zhang [7]) and on economic development (Zhang et al. [20]) but not the manipulation decision itself. Therefore, these works simply assumed that parents have perfect control over the gender of their children. Kim [13] relaxed this assumption in his analysis of the effects of gender selection on fertility by letting the gender of a child be random if parents do not engage in gender selection. Nevertheless, Kim's model still assumes that gender can be perfectly selected through the use of gender-selective abortion. Ebenstein [10] provided a model of fertility choice under the one-child policy in which parents incur a fine for having more than one child. In the model, parents have access to a gender-selection technology that is costly but perfectly effective. Ebenstein assumed that parents want a second child only if their first child is a girl and showed that gender imbalance worsens as the fine for excess-fertility increases or as the cost of gender-selection technology decreases.

In contrast to these earlier works, we build a model of manipulation in which success in gender selection is never guaranteed. We take this approach for two reasons. First and foremost, gender-selective abortion is illegal in China, and healthcare workers are not allowed to reveal the gender of a fetus to the parents. Therefore, parents attempting to engage in gender-selective abortion may fail to find a doctor who will acquiesce to their desires. Second, although gender-selective abortion has received the most attention from scholars, anecdotal evidence suggests that other methods of prenatal gender selection that are of controversial effectiveness are also widely practiced in China.[5] Consequently, we assume that parents have a choice between using a natural birth process, which produces a boy or a girl with equal probability, or manipulating (more precisely, attempting to manipulate) the birth process, which increases the likelihood of giving birth to a boy.

Given the intensity and pervasiveness of son-preference in China, a key question that arises is not why manipulation occurs at all but why it does not occur more

[5] For example, methods that rely on the timing of conception appear popular. However, studies on their effectiveness are inconclusive (Harlap [12]; Wilcox et al. [18]).

frequently.[6] One obvious answer is of course cost: manipulation is costly, so only those who value a son more than some threshold level will manipulate. However, cost cannot be the entire answer since it alone cannot satisfactorily explain why the gender ratio appears to have plateaued around year 2000, even as income continued to grow and the cost of manipulation presumably decreased through improvements in medical technology.[7] Indeed, both China and Korea's experiences seem to indicate that there may be a natural brake that dampens parents' desire to manipulate as gender imbalance grows and that the observed gender ratio is an outcome determined by the interaction between these two opposing forces.

To capture this aspect, we introduce a marriage market into our model as a countervailing force against manipulation. Specifically, we assume that, all things being equal, parents derive a higher intrinsic utility from a son than a daughter. At the same time, however, parents also receive a higher intrinsic utility from a child who successfully marries than from a child who fails to find a mate.[8] Since the likelihood of a son marrying successfully decreases as the proportion of males in the population increases, a high gender ratio tempers parents' desire to manipulate. Thus, a parent's utility from having a child is determined endogenously by a combination of the parent's intrinsic utility and the gender composition in the child's generation.

Because gender composition depends on the choices of all the parents, the parents' manipulation decisions are interdependent on each other. Therefore, we look for an equilibrium at the societal level, in which every parent's decision is optimal given the choices made by the other parents, and study the resulting level of gender imbalance. We show that an equilibrium level of gender imbalance, at which the intrinsic preference for a son is exactly balanced by the son's lowered prospect in the marriage market, always exists under the one-child policy and provide a simple numerical example. We then relax the one-child policy and allow parents to have up to two children, assuming that parents discount the utility from a second child. Focusing on the case in which every couple ends up having two children, we show that an equilibrium exists and provide comparative statics results. We also extend the example given for the one-child policy to the two-child case and numerically show that the gender imbalance can actually worsen under the two-child policy even when parents discount a second son more heavily than they would a daughter if they already have a son.

This may appear surprising at first since it seems as if gender imbalance should moderate under such scenario. For example, one may have expected that when parents have two chances at having a son, some of the parents who would have manipulated under the one-child policy would now be induced to use the natural birth process for their first child and then, depending on the outcome, decide whether to

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[6]See, for example, Arnold and Liu [1] for an empirical study on the intensity of son-preference in China.

[7]See footnote 1.

[8]Both anecdotal evidence and academic research indicate that Chinese parents go to extraordinary measures to make their son more marriageable. For example, Wei and Zhang [17] showed that parents increasing their savings in a "competitive manner" to make their son relatively more attractive in the marriage market accounts for about half of the rise in the household savings rate during 1990-2007.

manipulate their second parity. Such a switch in behavior would moderate the gender imbalance. However, if parents discount a second daughter more heavily than a second son, then the opposite switch can also occur. Because manipulation is not guaranteed to produce a son, some of the parents who would not have manipulated under the one-child policy may now manipulate their first parity (then decide whether to manipulate their second parity based on the outcome) to reduce the chance of ending up with two daughters. As we show in the example, additional boys who are produced by this preemptive switch can push the gender ratio above the one-child-policy level.

Our model is closely related to Bhaskar [3], who uses a similar framework to study the welfare implications of son-preference and gender selection in the presence of "marriage squeeze." However, although both models use the marriage market as a damper against gender selection, the nature of the equilibrium differs significantly in the two. Because parents are homogeneous in Bhaskar's model, equilibrium can only occur when every parent is exactly indifferent between manipulating or not. This makes the equilibrium inherently unstable since the slightest deviation from the equilibrium gender ratio triggers either everyone or no one to manipulate. In contrast, parents are heterogeneous in our model, and the additional utility they receive from having a son over a daughter, or vice versa, varies among the parents at any gender ratio. Thus, a gender ratio that is lower than the equilibrium induces more (but not all) manipulation while a higher ratio induces fewer (but not zero) manipulations, and the equilibrium gender ratio constitutes a stable point in this dynamics.

The remainder of the paper is organized as follows. Section 2 presents the model of parents' decision making under the one-child policy. Existence of a unique equilibrium and comparative statics results are established, and a numerical example is given. Although the parameter values for the example has been chosen to match China's current gender ratio at birth, their main purpose is to provide an example that will serve as a benchmark throughout the paper, and the qualitative conclusions of our paper are not affected by them. Section 3 considers the two-child policy. We first investigate initial responses to the policy by extending the one-child policy example to the two-child case and providing additional parameters of the model for which the gender ratio improves initially. We then show that a unique equilibrium always exists under the two-child policy and provide basic comparative statics results. We also show numerically that, for the parameters given in the previous example, the gender imbalance is made worse in equilibrium. Section 4 concludes. All proofs are given in the appendix.

2 One-child policy

We consider a large population Ω, which consists of parenting couples who give birth to exactly one child.[9] The children, who are all considered to be of the same genera-

[9]We will be assuming that Ω is uncountable. This technically requires that Ω is a part of a measure space $(\Omega, \mathscr{F}, \tilde{\mu})$, where $\mathscr{F}$ is a σ-field containing all the subsets of Ω of interest and $\tilde{\mu}$ is a measure with $\tilde{\mu}(\Omega) = 1$. So, for example, the terms "number" and "proportion" of couples in set A means $\tilde{\mu}(A)$,

tion, will grow up and either successfully marry someone in their generation or fail to find a mate. Let M_B^i and S_B^i denote couple i's intrinsic utility, net of any child-rearing cost, from giving birth to a boy who will eventually marry and who will remain single, respectively. Similarly, M_G^i and S_G^i denote the intrinsic utility from having a girl who will marry and who will remain single. To reflect the preference for sons, we assume that $M_B^i \geq M_G^i > S_B^i \geq S_G^i$ for all $i \in \Omega$.

Couples decide simultaneously whether to manipulate the birth process (m) or not (n) when they are having a child. If a couple does not manipulate, then the gender of the child is determined according to the natural process, which for simplicity is assumed to yield a boy or a girl with equal probability. If a couple manipulates, then the probability of giving birth to a boy is increased to $p_B \in \left(\frac{1}{2}, 1\right)$, while the probability of producing a girl is reduced to $p_G = 1 - p_B$.

Let $r = (r_B, r_G)$, where r_g is the proportion of gender g children in the population. We call r_B/r_G the gender ratio and r_G/r_B the inverse gender ratio. We assume that the probability of finding a mate for a child of given gender is

$$\pi_B(r) \; = \; \min\left\{\theta, \left(\frac{r_G}{r_B}\right)\theta\right\} \quad \text{and} \quad \pi_G(r) \; = \; \min\left\{\theta, \left(\frac{r_B}{r_G}\right)\theta\right\},$$

where $0 < \theta \leq 1$. When $r_G \leq r_B$,

$$r_B \times \pi_B(r) \; = \; r_B \times \left(\frac{r_G}{r_B}\right)\theta \; = \; r_G\theta \; = \; r_G \times \pi_G(r).$$

Similarly, $r_G \times \pi_G(r) = r_B \times \pi_B(r)$ when $r_B \leq r_G$. Thus, the mating probabilities satisfy the requirement that the expected number of boys and the expected number of girls who find their mate are the same. Moreover, the probabilities have the desirable feature that they increase (weakly) as the proportion of the opposite gender increases. When the gender ratio is one so that the boy and the girl populations are balanced, the probability of finding a mate is θ, and the expected fraction of the children who will fail to find a mate is $1-\theta$. Therefore, θ represents a friction parameter for the mating process.

Let $r^i = \left(r_B^i, r_G^i\right)$, where r_g^i is couple i's belief about the proportion of gender g children in the population. Then, given these beliefs, the couple's expected utility from giving birth to a gender g child is

$$u_g^i \; = \; M_g^i \pi_g(r^i) + S_g^i\left(1 - \pi_g(r^i)\right),$$

and the expected utility from taking action $a \in \{m, n\}$ is

$$u_a^i \; = \; p_{B|a} u_B^i + p_{G|a} u_G^i,$$

where $p_{g|a}$ is the probability of producing a child of gender g for action a.

assuming that A is measurable, and the term "every couple" means "except for couples in a set of measure zero."

It is easy to see that a couple will manipulate the birth process if and only if the expected utility from a son is greater than from a daughter:[10]

$$
\begin{aligned}
u_m^i > u_n^i &\iff p_B u_B^i + (1-p_B)u_G^i > \tfrac{1}{2}u_B^i + \tfrac{1}{2}u_G^i \\
&\iff \left(p_B - \tfrac{1}{2}\right)u_B^i > \left(p_B - \tfrac{1}{2}\right)u_G^i \\
&\iff u_B^i > u_G^i. \qquad (1)
\end{aligned}
$$

Let

$$
V_d^i = \frac{\left(\theta M_G^i + (1-\theta)S_G^i\right) - S_B^i}{\left(\theta M_B^i + (1-\theta)S_B^i\right) - S_B^i}.
$$

To clarify V_d^i, suppose that the couple could choose the gender of their child. Since θ is the highest possible mating probability, $\theta M_g^i + (1-\theta)S_g^i$ is the maximum possible expected utility obtainable from a child of gender g. Utility S_B^i is the maximin utility, the best possible utility the couple can guarantee themselves, and hence serves as a reference utility level. Therefore, V_d^i is the ratio of the maximum utility gain (relative to the reference utility) that is possible from choosing a girl to the maximum gain possible from choosing a boy. For this reason, we interpret V_d^i as an index of daughter value.

Given our assumption that $M_B^i \geq M_G^i > S_B^i \geq S_G^i$, the daughter-value index is always less than or equal to one. It equals one when $M_B^i = M_G^i$ and $S_B^i = S_G^i$, that is, when the couple is indifferent between a son and a daughter. It decreases as intrinsic utility from a daughter decreases or as intrinsic utility from a son increases.[11] The following lemma, obtained by further simplifying inequality (1), shows that parents will manipulate the birth process if and only if their daughter-value index is less than their belief for the inverse gender ratio.

Lemma 1. *We have*

$$
u_m^i > u_n^i \iff V_d^i < \frac{r_G^i}{r_B^i}.
$$

Proof. See Appendix. □

Following the rational expectations principle, we define an equilibrium in this model as a situation in which every couple's belief is consistent with the actions of the population. This requires not only that all the couples have the same belief but also that the actions of the couples given this belief generate an expected inverse gender

[10]We assume that when couples are indifferent between manipulating and not manipulating, they do not manipulate.

[11]It is easy to see that V_d^i is increasing in M_G^i and S_G^i and decreasing in M_B^i. Since

$$
\begin{aligned}
\operatorname{sign}\left(\partial V_d^i \big/ \partial S_B^i\right) &= \operatorname{sign}\left(-\left(\theta M_B^i + (1-\theta)S_B^i - S_B^i\right) + \theta\left(\theta M_G^i + (1-\theta)S_G^i - S_B^i\right)\right) \\
&= \operatorname{sign}\left(-M_B^i + \theta M_G^i + (1-\theta)S_G^i\right) = (-),
\end{aligned}
$$

it is also decreasing in S_B^i.

ratio equal to the belief. An immediate consequence of Lemma 1 is that the inverse gender ratio cannot be greater than one in equilibrium. If it were, then everyone will choose to manipulate because the daughter-value index is always less than or equal to one. However, this in turn will induce the expected inverse gender ratio to be less than one, leading to a contradiction.

Before defining the equilibrium formally, we first normalize the utilities by setting $M_B^i = 1$ and $S_G^i = 0$ for all the couples so that each couple can be identified with their utility profile, $\left(S_B^i, M_G^i\right)$.[12] In the remainder of the paper, we assume that the utility profiles are distributed continuously in

$$\Omega \;=\; \left\{(S_B, M_G) \in [0,1]\times[0,1]:\; M_G > S_B\right\},$$

which is the upper triangular area in figure 1.

Our normalization simplifies the daughter-value index to $\left(\theta M_G^i - S_B^i\right)/\left(\theta - \theta S_B^i\right)$. Thus, the condition for manipulation given in Lemma 1 can be restated as a linear inequality in two variables:

$$u_m^i \;>\; u_n^i \iff \frac{\theta M_G^i - S_B^i}{\theta - \theta S_B^i} \;<\; \frac{r_G^i}{r_B^i} \iff M_G^i \;<\; \frac{r_G^i}{r_B^i} + \left(\frac{1}{\theta} - \frac{r_G^i}{r_B^i}\right) S_B^i. \tag{2}$$

Condition (2) implies that, given any common belief $r = (r_B, r_G)$, the set of couples who will manipulate is given by the area

$$A_m(r) \;=\; \left\{(S_B, M_G) \in \Omega:\; M_G \;<\; \frac{r_G}{r_B} + \left(\frac{1}{\theta} - \frac{r_G}{r_B}\right) S_B\right\},$$

and the set of non-manipulators is given by

$$A_n(r) \;=\; \left\{(S_B, M_G) \in \Omega:\; M_G \;\geq\; \frac{r_G}{r_B} + \left(\frac{1}{\theta} - \frac{r_G}{r_B}\right) S_B\right\}.$$

Figure 1 illustrates these sets using the parameter values from Example 1 below.[13] The manipulators occupy the region closer to the main diagonal line because that is where the utility from a single son is close to the utility from a married daughter, which indicates low daughter value.

In general, the relative sizes of the two areas do not necessarily reflect the proportions of the manipulators and the non-manipulators since the proportions also

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[12]This is without loss of generality since a couple's action choice depends only on their own daughter-value index and their belief. To see this, let $\tilde{M}_g^i$ and $\tilde{S}_g^i$ be the original utilities, and let $M_g^i = \left(\tilde{M}_g^i - \tilde{S}_G^i\right)/\left(\tilde{M}_B^i - \tilde{S}_G^i\right)$ and $S_g^i = \left(\tilde{S}_g^i - \tilde{S}_G^i\right)/\left(\tilde{M}_B^i - \tilde{S}_G^i\right)$ be the normalized utilities. Then

$$\frac{\theta M_G^i + (1-\theta) S_G^i - S_B^i}{\theta M_B^i + (1-\theta) S_B^i - S_B^i} = \frac{\theta\left(\tilde{M}_G^i - \tilde{S}_G^i\right) + (1-\theta)\left(\tilde{S}_G^i - \tilde{S}_G^i\right) - \left(\tilde{S}_B^i - \tilde{S}_G^i\right)}{\theta\left(\tilde{M}_B^i - \tilde{S}_G^i\right) + (1-\theta)\left(\tilde{S}_B^i - \tilde{S}_G^i\right) - \left(\tilde{S}_B^i - \tilde{S}_G^i\right)} = \frac{\theta \tilde{M}_G^i + (1-\theta)\tilde{S}_G^i - \tilde{S}_B^i}{\theta \tilde{M}_B^i + (1-\theta)\tilde{S}_B^i - \tilde{S}_B^i}.$$

[13]Since $M_G \leq 1$, the maximum possible value for S_B in $A_n(r)$ is found by

$$\frac{r_G}{r_B} + \left(\frac{1}{\theta} - \frac{r_G}{r_B}\right) S_B \;\leq\; 1 \iff S_B \;\leq\; \frac{1 - r_G/r_B}{1/\theta - r_G/r_B}.$$

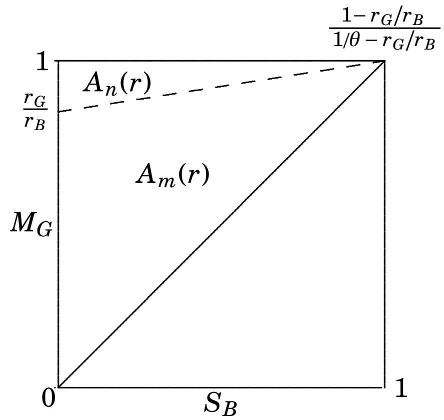

Figure 1: Manipulators and non-manipulators.
$\theta = 1$ and $r_G/r_B = 0.844$.

depend on the distribution of the utility profiles among the population. That is, letting f be the joint density function for (S_B, M_G) and $\tilde{\mu}(A)$ be the mass of couples in region A, the proportion of couples choosing action a is given by

$$\mu_a(r) \;=\; \tilde{\mu}(A_a(r)) \;=\; \int\limits_{A_a(r)} f(S_B, M_G)\, d(S_B, M_G).$$

The expected proportion of the boys in the next generation is

$$\rho_B(r) \;=\; p_B\mu_m(r) + \tfrac{1}{2}\mu_n(r) \;=\; p_B\mu_m(r) + \tfrac{1}{2}(1-\mu_m(r)) \;=\; \tfrac{1}{2} + \left(p_B - \tfrac{1}{2}\right)\mu_m(r),$$

and the expected proportion of the girls is

$$\rho_G(r) \;=\; 1 - \rho_B(r) \;=\; \tfrac{1}{2} - \left(p_B - \tfrac{1}{2}\right)\mu_m(r).$$

Definition 1. An equilibrium under the one-child policy is a gender composition $r^* = \left(r_B^*, r_G^*\right)$ such that

$$r^* \;=\; \left(\rho_B(r^*), \rho_G(r^*)\right).$$

That is, in equilibrium r^*, if every couple believes that the gender composition will be r^* and chooses their optimal action accordingly, their choices actually induce a gender composition equal to their belief, in expected terms. The following theorem shows that a unique equilibrium exists if the density of utility profiles is positive everywhere. The theorem also shows that the equilibrium gender imbalance grows as the probability of successful manipulation increases. Interestingly, the imbalance also worsens if the friction in the mating process increases. This occurs because a drop in marriage probability lowers the expected utility from a daughter more than the utility from a son.

Theorem 2. *Suppose $f(S_B, M_G) > 0$ for all $(S_B, M_G) \in \Omega$. Then a unique equilibrium $r^* = \left(r_B^*, r_G^*\right)$ exists for each $p_B \in \left(\frac{1}{2}, 1\right)$ and $\theta \in (0, 1]$. Moreover, the equilibrium gender ratio is always greater than one and is increasing in p_B and decreasing in θ.*

Proof. See Appendix. □

The following gives an example of an equilibrium under the one-child policy that will serve as the benchmark throughout the paper.

Example 1. Suppose that the utility profiles are distributed uniformly in Ω. Then $f(S_B, M_G) = 2$ for all (S_B, M_G), and $\mu_a(r)$ is simply twice the area of $A_a(r)$. Therefore,

$$\mu_m(r) \;=\; 1 - \mu_n(r) \;=\; 1 - \frac{\left(1 - r_G/r_B\right)^2}{1/\theta - r_G/r_B},$$

and the equilibrium condition becomes

$$r_B^* \;=\; \frac{1}{2} + \left(p_B - \frac{1}{2}\right)\left(1 - \frac{\left(1 - \frac{1-r_B^*}{r_B^*}\right)^2}{\frac{1}{\theta} - \frac{1-r_B^*}{r_B^*}}\right).$$

Let $\theta = 1$ and $p_B = 0.55$. Then the above equation simplifies to

$$r_B^* \;=\; \frac{1}{2} + \frac{0.05(1 - r_B^*)}{r_B^*} \iff (r_B^*)^2 - 0.45 r_B^* - 0.05 \;=\; 0.$$

Numerically solving for the root of this equation yields $r_B^* = 0.542$, which means that the equilibrium gender and inverse gender ratios are $r_B^*/r_G^* = 0.542/0.458 = 1.184$ and $r_G^*/r_B^* = 0.844$, which closely match 1.181 gender ratio at birth recorded by China's 2010 census.[14] The corresponding $A_m(r^*)$ and $A_n(r^*)$ are depicted in figure 1. □

Before proceeding to the analysis of the two-child policy, we make the following remark on the interpretation of our model.

Remark. Although we have motivated our formulation of the endogenous utilities by assuming that parents have consideration for their children's ability to marry, we need not impose this particular assumption on the model. Concentrating on the case where the gender ratio is greater than one, we can interpret our model as one in which the cost of raising a son is increasing in the gender ratio and leave aside the exact source of the additional cost, though it is presumably from an increased competitive pressure faced by males. To see this, let $\phi_g^i \;=\; M_g^i \theta + S_g^i(1-\theta)$ be the net utility from a gender g child, which does not depend on the gender ratio. Then, assuming $r_B \geq r_G$, we have

$$u_G^i \;=\; M_G^i \pi_G^i + S_G^i(1 - \pi_G^i) \;=\; M_G^i \theta + S_G^i(1-\theta) \;=\; \phi_G^i$$

and

$$\begin{aligned} u_B^i \;&=\; M_B^i \pi_B^i + S_B^i(1 - \pi_B^i) \;=\; M_B^i\left(\frac{r_G}{r_B}\theta\right) + S_B^i\left(1 - \frac{r_G}{r_B}\theta\right) \\ &=\; \left[M_B^i \theta + S_B^i(1-\theta)\right] - \underbrace{\theta(M_B^i - S_B^i)\left[1 - \left(\frac{r_B}{r_G}\right)^{-1}\right]}_{\equiv\, \psi^i\left(\frac{r_B}{r_G}\right)} \\ &=\; \phi_B^i - \psi^i\left(\frac{r_B}{r_G}\right), \end{aligned}$$

where the additional cost of raising a son, ψ^i, is increasing and concave in r_B/r_G. □

[14] See footnote 1.

3 Two-child policy

Suppose that the one-child policy is relaxed to allow couples to have up to two children. To focus on the effect of the policy change on the incentives to manipulate, we assume that the (net) utility from a second child is always non-negative so that every couple will have a second child.[15] Setting aside the timing of the policy announcement for the moment, we assume that the sequence of couples' decision making is as follows. All the couples simultaneously decide whether to manipulate the birth process for their first child. After observing the gender of their own child but not those of others, the couples simultaneously decide whether to manipulate or not for their second child. The stage in which decisions regarding the k-th child is being made is referred to as stage k. The action taken in stage k is denoted a_k, and the gender of the k-th child is denoted g_k.

We model the trade-offs in the second-stage decision making by assuming that the utility gained from a second child is discounted. Specifically, we let $\delta_{g_2|g_1} \in (0,1]$ be the discount factor on the intrinsic utility from a second child of gender g_2 when the gender of the first child is g_1. To reflect the preference for sons and at the same time tilt the preference toward children of mixed gender, we further assume $\delta_{B|G} \geq \delta_{G|B} > \delta_{B|B} \geq \delta_{G|G}$. In particular, couples discount a second child more heavily if the child is of the same gender as their first child. In addition, they discount a second daughter (weakly) more heavily than a second son.

We also assume that a couple's belief about the gender ratio does not change after the birth of their first child. Thus, couple i's expected utility from having a second child who is of gender g_2 is

$$u^i_{g_2|g_1} \;=\; \delta_{g_2|g_1} M^i_{g_2} \pi^i_{g_2} + \delta_{g_2|g_1} S^i_{g_2} \left(1 - \pi^i_{g_2}\right) \;=\; \delta_{g_2|g_1} u^i_{g_2},$$

and the overall expected utility from the two children is

$$u^i_{g_1 g_2} \;=\; u^i_{g_1} + u^i_{g_2|g_1} \;=\; u^i_{g_1} + \delta_{g_2|g_1} u^i_{g_2}.$$

The couple's expected utility in stage 2 from taking action a_2, given g_1, is

$$u^i_{a_2|g_1} \;=\; p_{B|a_2} u^i_{B|g_1} + p_{G|a_2} u^i_{G|g_1} \;=\; p_{B|a_2} \left(\delta_{B|g_1} u^i_B\right) + p_{G|a_2} \left(\delta_{G|g_1} u^i_G\right),$$

and the expected utility from taking action a_1 in stage 1 and then a_B in the second stage if the gender of the first child is B and a_G if it is a girl is given by

$$u^i_{a_1 a_B a_G} \;=\; u^i_{a_1} + p_{B|a_1} u^i_{a_B|B} + p_{G|a_1} u^i_{a_G|G}.$$

Although we are ultimately interested in the equilibrium behavior under this two-child policy framework, we begin by investigating how the population will respond initially to the policy. To that end, Subsection 3.1 assumes that the policy is

[15] We make this assumption to facilitate the analysis. However, unless there is a reason to suspect that parents are more likely to stop at first parity after having a daughter than a son, removing this assumption should not change our central conclusion that the gender ratio may not improve under the two-child policy.

announced after all the couples already had their first child and that the couples react without changing their belief about the gender ratio. This allows us to restrict attention to the optimal second-stage strategy, which is given in Lemma 3.[16] Theorem 4 establishes the condition under which gender imbalance will improve initially, and Example 2 provides the parameters for which this happens.

Subsection 3.2 then assumes that the policy is implemented before the couples had their first child and uses the optimal second-stage strategy identified in Subsection 3.1 to investigate the equilibrium. Lemma 5 characterizes the optimal overall strategy, and Theorem 6 establishes the existence of an equilibrium, which is unique, and gives basic comparative statics results. Example 3 shows that the gender ratio can worsen under the two-child policy if the value of a second daughter is sufficiently lower than the value of a second son.

3.1 Initial response

Suppose that the two-child policy is announced after the couples already had their first child. Then a couple's only remaining decision is whether to manipulate the birth process for their second child. Let

$$V^i_{d|g_1} = \frac{\delta_{G|g_1}\left(\theta M^i_G + (1-\theta) S^i_G\right) - \delta_{B|g_1} S^i_B}{\delta_{B|g_1}\left(\theta M^i_B + (1-\theta) S^i_B\right) - \delta_{B|g_1} S^i_B}$$

be an index of discounted daughter value. The following lemma shows that a couple's optimal action in the second stage may depend on the gender of the first child but not on the action taken in the first stage. Moreover, the optimal action is determined by whether the appropriate discounted daughter-value index is smaller or larger than the couple's belief about the inverse gender ratio.

Lemma 3. *Given belief r^i, couple i's optimal strategy in the second stage takes the form $a_B a_G$, where a_{g_1} is the action choice if their first child is of gender g_1. Moreover, it is given by*

$$a_B a_G = \begin{cases} mm & \text{if } V^i_{d|B} < \frac{r^i_G}{r^i_B} \\ nm & \text{if } V^i_{d|G} < \frac{r^i_G}{r^i_B} \le V^i_{d|B} \\ nn & \text{if } V^i_{d|G} \ge \frac{r^i_G}{r^i_B}. \end{cases}$$

Proof. See Appendix. □

As the lemma shows, $a_B a_G = mn$ is never optimal. This follows from our assumption on the discount factors, which favors children of different genders. For example, any couple whose preference for sons is high enough to manipulate even after having one boy will certainly manipulate if they had a girl instead since their desire for a boy is reinforced by their desire for a child of different gender. Therefore, mm can be optimal but not mn.

[16] By strategy, we mean a plan of actions that specifies what a couple will do in each contingency.

To graphically represent the optimal second-stage strategies for the population, we again set $M_B^i = 1$ and $S_G^i = 0$ and obtain[17]

$$V_{d|g_1}^i < \frac{r_G^i}{r_B^i} \iff M_G^i < \frac{\delta_{B|g_1}}{\delta_{G|g_1}}\left[\frac{r_G^i}{r_B^i} + \left(\frac{1}{\theta} - \frac{r_G^i}{r_B^i}\right)S_B^i\right]. \tag{3}$$

Let $A_{a_B a_G}(r)$ be the set of couples whose optimal second-stage strategy given common belief r is $a_B a_G$. Then Lemma 3 yields the following.

$$\begin{aligned}
A_{mm}(r) &= \left\{(S_B, M_G) \in \Omega : \, M_G < \frac{\delta_{B|B}}{\delta_{G|B}}\left[\frac{r_G}{r_B} + \left(\frac{1}{\theta} - \frac{r_G}{r_B}\right)S_B\right]\right\}, \\
A_{nm}(r) &= \left\{(S_B, M_G) \in \Omega : \, \frac{\delta_{B|B}}{\delta_{G|B}}\left[\frac{r_G}{r_B} + \left(\frac{1}{\theta} - \frac{r_G}{r_B}\right)S_B\right] \leq M_G \right. \\
&\qquad \left. < \frac{\delta_{B|G}}{\delta_{G|G}}\left[\frac{r_G}{r_B} + \left(\frac{1}{\theta} - \frac{r_G}{r_B}\right)S_B\right]\right\}, \text{ and} \\
A_{nn}(r) &= \left\{(S_B, M_G) \in \Omega : \, M_G \geq \frac{\delta_{B|G}}{\delta_{G|G}}\left[\frac{r_G}{r_B} + \left(\frac{1}{\theta} - \frac{r_G}{r_B}\right)S_B\right]\right\}.
\end{aligned}$$

Since $\frac{\delta_{B|B}}{\delta_{G|B}} \leq 1 \leq \frac{\delta_{B|G}}{\delta_{G|G}}$, inequalities (2) and (3) imply that $A_{mm}(r)$ is a subset of $A_m(r)$ and $A_{nn}(r)$ is a subset of $A_n(r)$. That is, of the couples who manipulated under the one-child policy, those with strong enough son-preference will manipulate again whether they already have a son or not, while the remaining couples will manipulate if and only if they have a daughter. Similarly, of the couples who did not manipulate under the one-child policy, those with high enough daughter value will never manipulate in the second stage, while the remainder will manipulate if they have a daughter. To summarize, let $\tilde{A}_{a_1 a_B a_G}(r) = A_{a_1}(r) \cap A_{a_B a_G}(r)$ be the set of couples who chose a_1 under the one-child policy and whose optimal strategy for their second child is $a_B a_G$.[18] Then Lemmas 1 and 3 partition the population into $\tilde{A}_{mmm}(r)$, $\tilde{A}_{mnm}(r)$, $\tilde{A}_{nnm}(r)$, and $\tilde{A}_{nnn}(r)$, with

$$A_m(r) = \tilde{A}_{mmm}(r) \cup \tilde{A}_{mnm}(r) \quad \text{and} \quad A_n(r) = \tilde{A}_{nnm}(r) \cup \tilde{A}_{nnn}(r).$$

Figure 2 depicts these regions when the couples maintain the equilibrium belief r^* given in Example 1.[19] As in figure 1, the area below the dashed line represents the couples who manipulated under the one-child policy, and the area above represents those who did not. The solid lines further divide the population according to their optimal second-stage strategy. Figure 2 assumes $\delta_{B|G} = \delta_{G|B} = 1$ so that son-preferences are not exaggerated. The figure further assumes $\delta_{B|B} = 0.8$ and $\delta_{G|G} = 0.5$. As the

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[17]We have

$$V_{d|g_1}^i < \frac{r_G^i}{r_B^i} \iff \frac{\delta_{G|g_1}\theta M_G^i - \delta_{B|g_1} S_B^i}{\delta_{B|g_1}\left(\theta - \theta S_B^i\right)} < \frac{r_G^i}{r_B^i} \iff M_G^i < \frac{\delta_{B|g_1}}{\delta_{G|g_1}}\left[\frac{r_G^i}{r_B^i} + \left(\frac{1}{\theta} - \frac{r_G^i}{r_B^i}\right)S_B^i\right].$$

[18]We use tildes in the notation here to denote that a_1 is not necessarily the optimal first-stage action under the two-child policy since it was chosen prior to the announcement of the policy.

[19]The figure is discussed further in Example 2.

figure illustrates, there are no couples choosing nn in this case; that is, every couple manipulates if they had a girl.

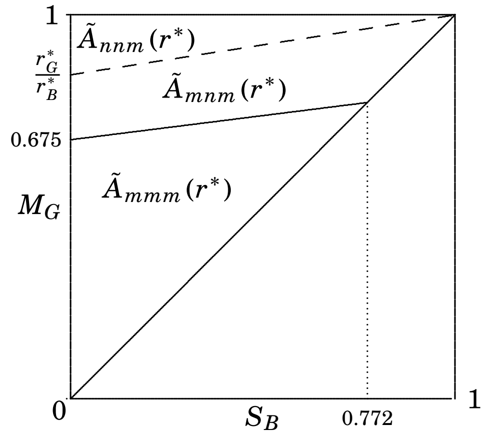

Figure 2: Optimal strategies in the second stage.
$\theta = 1$, $\delta_{B|G} = \delta_{G|B} = 1$, $\delta_{B|B} = 0.8$, $\delta_{G|G} = 0.5$, and $r_G^*/r_B^* = 0.844$.

The ex-ante probability of having a boy as a second child is p_B for couples in $\tilde{A}_{mmm}(r)$ and $\frac{p_B}{2} + (1-p_B)p_B < p_B$ for those in $\tilde{A}_{mnm}(r)$.[20] The probability is $\frac{1}{2}$ for couples in $\tilde{A}_{nnn}(r)$ and $\frac{1}{4} + \frac{p_B}{2} > \frac{1}{2}$ for those in $\tilde{A}_{nnm}(r)$. Therefore, letting $\mu_{a_1 a_B a_G}(r) = \tilde{\mu}\left(\tilde{A}_{a_1 a_B a_G}(r)\right)$, the expected proportion of boys among the second children is

$$\begin{aligned}\rho_{B_2}(r) \;=\; & (p_B)\,\mu_{mmm}(r) + \left(\tfrac{p_B}{2} + (1-p_B)p_B\right)\mu_{mnm}(r) \\ & + \left(\tfrac{1}{2}\right)\mu_{nnn}(r) + \left(\tfrac{1}{4} + \tfrac{p_B}{2}\right)\mu_{nnm}(r). \end{aligned} \tag{4}$$

Thus, whether the gender ratio improves relative to the one-child policy depends on the number of couples choosing nm who manipulated under the one-child policy versus those who did not.

Theorem 4. *Suppose that the two-child policy is announced after the couples had their first child, and the couples react to the policy without changing their belief about the gender ratio. Then the expected gender ratio decreases relative to the one child-policy if and only if* $p_B \mu_{mnm}(r^*) \;>\; \frac{1}{2}\mu_{nnm}(r^*)$.

This result can be interpreted in the following way. A couple using strategy mnm will reduce the expected gender ratio among the second borns if and only they have a boy as a first child (and hence not manipulate in the second stage), which is a probability p_B event. In contrast, a couple engaging in nnm will increase the ratio

[20] Let $p_{B_2|a_1 a_B a_G} = p_{B|a_1} p_{B|a_B} + p_{G|a_1} p_{B|a_G}$ be the ex-ante probability of having a boy as a second child for a couple in $\tilde{A}_{a_1 a_B a_G}(r)$. Then

$$\begin{aligned} p_{B_2|mmm} \;&=\; p_B(p_B) + (1-p_B)p_B \;=\; p_B \;>\; p_B\left(\tfrac{1}{2}\right) + (1-p_B)p_B \;=\; p_{B_2|mnm}, \\ \text{and} \quad p_{B_2|nnn} \;&=\; \tfrac{1}{2}\tfrac{1}{2} + \tfrac{1}{2}\tfrac{1}{2} \;=\; \tfrac{1}{2} \;<\; \tfrac{1}{2}\tfrac{1}{2} + \tfrac{1}{2}p_B \;=\; p_{B_2|nnm}. \end{aligned}$$

if and only if they have a girl (and hence manipulate in the second stage), which occurs with probability $\frac{1}{2}$. Thus, the expected gender ratio will improve if and only if $p_B \mu_{mnm}(r^*) > \frac{1}{2}\mu_{nnm}(r^*)$.

The following example, which continues Example 1, provides one set of parameter values for which the expected gender ratio improves initially.

Example 2. Starting from equilibrium $r^* = (0.542, 0.458)$ found in Example 1, suppose that the two-child policy is announced, and the parents react without changing their belief that the inverse gender ratio will be 0.844. Suppose further that the discount factors are $\delta_{B|G} = \delta_{G|B} = 1$, $\delta_{B|B} = 0.8$, and $\delta_{G|G} = 0.5$. Then

$$\begin{aligned}
\tilde{A}_{mmm}\left(r^*\right) &= \left\{(S_B, M_G) \in \Omega : \; M_G \; < \; 0.675 + 0.125 S_B\right\}, \\
\tilde{A}_{mnm}\left(r^*\right) &= A_m\left(r^*\right) \setminus \tilde{A}_{mmm}\left(r^*\right), \\
\tilde{A}_{nnn}\left(r^*\right) &= \left\{(S_B, M_G) \in \Omega : \; M_G \; \geq \; 1.689 + 0.311 S_B\right\} \; = \; \emptyset, \text{ and} \\
\tilde{A}_{nnm}\left(r^*\right) &= A_n\left(r^*\right) \setminus \tilde{A}_{nnn}\left(r^*\right) \; = \; A_n\left(r^*\right),
\end{aligned}$$

which are illustrated in figure 2. Calculating the mass of each set yields $\mu_{mmm}(r^*) = 0.521$, $\mu_{mnm}(r^*) = 0.323$, $\mu_{nnn}(r^*) = 0$, and $\mu_{nnm}(r^*) = 0.156$. Since

$$(0.55)\mu_{mnm}\left(r^*\right) \; = \; 0.178 \; > \; 0.078 \; = \; \tfrac{1}{2}\mu_{nnm}\left(r^*\right),$$

Theorem 4 implies that the expected gender ratio among the second borns will improve.

Indeed, we have

$$\begin{aligned}
r_{B_2} &= p_B \mu_{mmm}\left(r^*\right) + \left(\tfrac{p_B}{2} + (1-p_B)p_B\right)\mu_{mnm}\left(r^*\right) + \tfrac{1}{2}\mu_{nnn}\left(r^*\right) + \left(\tfrac{1}{4} + \tfrac{p_B}{2}\right)\mu_{nnm}\left(r^*\right) \\
&= (0.55)(0.521) + (0.523)(0.323) + (.5)(0) + (0.525)(0.156) \; = \; 0.537.
\end{aligned}$$

Thus, the expected gender ratios among the second borns and all the children are, respectively,

$$\frac{r_{B_2}}{r_{G_2}} = \frac{0.537}{0.463} = 1.161 \quad \text{and} \quad \frac{r_B^* + r_{B_2}}{r_G^* + r_{G_2}} = \frac{0.542 + 0.537}{0.458 + 0.463} = 1.173.$$

Therefore, we have an improvement in the gender imbalance compared to the one-child policy. □

3.2 Equilibrium response

Now, suppose that the two-child policy is announced before the couples have their first child. Equivalently, we may suppose that the policy is being applied to a new generation of couples that has the same distribution of utility profiles as the original population. Because Lemma 3 shows that the optimal action in the second stage depends on the first-stage action only through the realized gender of the first child, the timing of the policy announcement does not affect the second-stage decision. However, a couple's first-stage decision must now take into account its consequences on the second stage. That is, given second-stage strategy $a_B a_G$,

$$\begin{aligned} u^i_{ma_Ba_G} > u^i_{na_Ba_G} &\iff p_B\left(u^i_B + u^i_{a_B|B}\right) + (1-p_B)\left(u^i_G + u^i_{a_G|G}\right) \\ &\qquad > \tfrac{1}{2}\left(u^i_B + u^i_{a_B|B}\right) + \tfrac{1}{2}\left(u^i_G + u^i_{a_G|G}\right) \\ &\iff u^i_B + u^i_{a_B|B} > u^i_G + u^i_{a_G|G}. \end{aligned} \tag{5}$$

Therefore, a couple will manipulate in the first stage if and only if their expected utility from having a boy plus the utility from the action they will take after having a boy is greater than their expected utility from a girl and the action they will take after having a girl. Simplifying inequality (5) yields the following result.

Lemma 5. *Suppose that, given belief r^i, $a_B a_G$ is the optimal second-stage strategy for couple i. Then $ma_B a_G$ is the overall optimal strategy if*

$$M^i_G < \frac{1 + p_{B|a_B}\delta_{B|B} - p_{B|a_G}\delta_{B|G}}{1 + p_{G|a_G}\delta_{G|G} - p_{G|a_B}\delta_{G|B}} \left[\frac{r^i_G}{r^i_B} + \left(\frac{1}{\theta} - \frac{r^i_G}{r^i_B}\right) S^i_B\right].$$

Otherwise, $na_B a_G$ is optimal. Moreover, mnn is never optimal if $\delta_{B|G} = 1$, and nmm is never optimal if $\delta_{G|B} = 1$.

Proof. See Appendix. □

If $\delta_{B|G} = 1$, the intrinsic utility from a boy first child is the same as the utility from having a boy as a second child after having a daughter. Therefore, any couple whose preference for a son is weak enough that they will not manipulate even after having a daughter will not manipulate their first birth either. Similarly, if $\delta_{G|B} = 1$, the intrinsic utility from a girl first child is the same as the utility from having a girl as a second child after having a boy. Thus, couples whose preference for a girl is low enough to manipulate even after already having a boy will also manipulate their first birth.

Given common belief r, let $A_{a_1 a_B a_G}(r)$ be the set of couples whose optimal strategy is $a_1 a_B a_G$. Then

$$\begin{aligned} A_{ma_Ba_G}(r) &= \Bigg\{(S_B, M_G) \in A_{a_Ba_G} : \\ &\qquad M_G < \frac{1 + p_{B|a_B}\delta_{B|B} - p_{B|a_G}\delta_{B|G}}{1 + p_{G|a_G}\delta_{G|G} - p_{G|a_B}\delta_{G|B}} \left[\frac{r_G}{r_B} + \left(\frac{1}{\theta} - \frac{r_G}{r_B}\right) S_B\right]\Bigg\} \\ A_{na_Ba_G}(r) &= \Bigg\{(S_B, M_G) \in A_{a_Ba_G} : \\ &\qquad M_G \geq \frac{1 + p_{B|a_B}\delta_{B|B} - p_{B|a_G}\delta_{B|G}}{1 + p_{G|a_G}\delta_{G|G} - p_{G|a_B}\delta_{G|B}} \left[\frac{r_G}{r_B} + \left(\frac{1}{\theta} - \frac{r_G}{r_B}\right) S_B\right]\Bigg\}. \end{aligned}$$

Lemmas 3 and 5 partition the population into six regions, $A_{mmm}(r)$, $A_{nmm}(r)$, $A_{mnm}(r)$, $A_{nnm}(r)$, $A_{mnn}(r)$, and $A_{nnn}(r)$, some of which may be empty. Let $\mu_{a_1 a_B a_G}(r)$ be the mass of couples in $A_{a_1 a_B a_G}(r)$. Calculating the expected proportion of boys using the probabilities given in table 1 in the appendix yields

$$\begin{aligned}\rho_B(r) \;=\; & \tfrac{1}{2}\Big[(2p_Bp_B + p_Bp_G + p_Gp_B)\,\mu_{mmm}(r) + \big(p_B + \tfrac{p_G}{2} + \tfrac{p_B}{2}\big)\,\mu_{nmm}(r) \\ & + \big(p_B + \tfrac{p_B}{2} + p_Gp_B\big)\,\mu_{mnm}(r) + \big(\tfrac{1}{2} + \tfrac{1}{4} + \tfrac{p_B}{2}\big)\,\mu_{nnm}(r) \\ & + \big(p_B + \tfrac{p_B}{2} + \tfrac{p_G}{2}\big)\,\mu_{mnn}(r) + \big(\tfrac{1}{2} + \tfrac{1}{4} + \tfrac{1}{4}\big)\,\mu_{nnn}(r)\Big].\end{aligned}$$

Equilibrium under the two-child policy is defined similarly to the one-child case.

Definition 2. An equilibrium under the two-child policy is a gender composition $r^{**} = \left(r_B^{**},\, r_G^{**}\right)$ such that

$$r^{**} \;=\; \left(\rho_B\left(r^{**}\right),\; \rho_G\left(r^{**}\right)\right).$$

The following theorem shows that a unique equilibrium always exists and provides comparative statics results.

Theorem 6. *Suppose $f(S_B,M_G) > 0$ for all $(S_B,M_G) \in \Omega$ and $\delta_{B|G} = \delta_{G|B} = 1$. Then a unique equilibrium $r^{**} = \left(r_B^{**}, r_G^{**}\right)$ exists for each $p_B \in \left(\frac{1}{2},1\right)$, $\theta \in (0,1]$, and discount factors $1 > \delta_{B|B} \geq \delta_{G|G} > 0$. Moreover, the equilibrium gender ratio is increasing in $\delta_{B|B}$ and decreasing in $\delta_{G|G}$.*

Proof. See Appendix. □

As the above result shows, the equilibrium expected gender imbalance worsens as the discount value $\delta_{B|B}$ increases and as $\delta_{G|G}$ decreases. In fact, the following example illustrates that the expected gender ratio may be higher under the two-child policy than the one-child policy if $\delta_{G|G}$ is sufficiently lower than $\delta_{B|B}$.

Example 3. Continuing Example 2, suppose that the two-child policy is announced before the couples have their first child. Numerically finding the solution to the equilibrium condition in Definition 2 yields $r_B^{**} = 0.543$. To verify that this is indeed the equilibrium proportion of boys, we first note that

$$A_{nn}\left(r^{**}\right) \;=\; \{(S_B, M_G) \in \Omega :\; M_G \;\geq\; 1.681 + 0.319 S_B\} \;=\; \emptyset.$$

Thus, $A_{nnn}(r^{**}) = A_{mnn}(r^{**}) = \emptyset$. Next,

$$\begin{aligned} A_{mmm}\left(r^{**}\right) &= \{(S_B, M_G) \in \Omega :\; M_G \;<\; 0.672 + 0.128 S_B\}, \\ A_{nmm}\left(r^{**}\right) &= \emptyset \text{ by Lemma 5}, \\ A_{nnm}\left(r^{**}\right) &= \{(S_B, M_G) \in \Omega : 0.672 + 0.128 S_B \;\leq\; M_G \;<\; 0.985 + 0.183 S_B\}, \text{ and} \\ A_{nnm}\left(r^{**}\right) &= \{(S_B, M_G) \in \Omega :\; M_G \;\geq\; 0.985 + 0.187 S_B\}. \end{aligned}$$

These regions are depicted in figure 3, and calculating the areas of the non-empty regions yields $\mu_{mmm}(r^{**}) = 0.518$, $\mu_{mnm}(r^{**}) = 0.481$, and $\mu_{nnm}(r^{**}) = 0.001$. Thus,

$$\begin{aligned} \rho_B\left(r^{**}\right) \;&=\; \frac{(2p_Bp_B + 2p_Bp_G)\,\mu_{mmm}(r^*) + \left(\frac{3p_B}{2} + p_Gp_B\right)\mu_{mnm}(r^*) + \left(\frac{3}{4} + \frac{p_B}{2}\right)\mu_{nnm}(r^*)}{2} \\ &=\; \frac{(1.1)(0.518) + (1.073)(0.481) + (1.025)(0.001)}{2} \;=\; 0.543 \;=\; r_B^{**}. \end{aligned}$$

The corresponding expected gender ratio is

$$\frac{r_B^{**}}{r_G^{**}} = \frac{0.543}{0.457} = 1.190 > 1.184 = \frac{r_B^*}{r_G^*}.$$

Therefore, the equilibrium gender ratio is worse under the two-child policy than the one-child policy. □

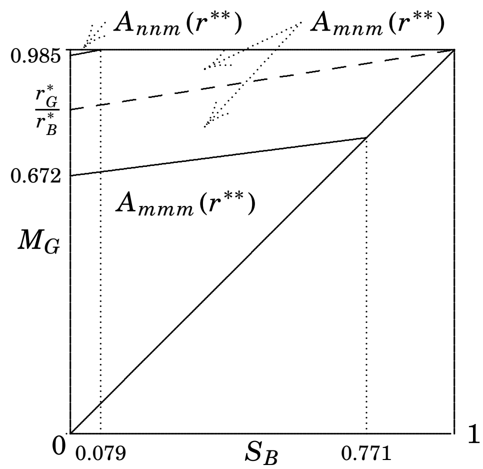

Figure 3: Equilibrium under the two-child policy.
$\theta = 1$, $\delta_{B|G} = \delta_{G|B} = 1$, $\delta_{B|B} = 0.8$, $\delta_{G|G} = 0.5$, and $r_G^{**}/r_B^{**} = 0.840$.

Figure 3 illustrates the equilibrium behaviors found in Example 3. The dashed line again represents the border between manipulation and non-manipulation under the one-child policy. In contrast to figure 2, the boundary separating couples choosing *nnm* and *mnm* is above the dashed line, making the intersection of $A_n(r^*)$ and $A_{mnm}(r^{**})$ nonempty. This means that some of the couples who would not have manipulated under the one-child policy will now manipulate their first birth and then use *nm* in the second stage, instead of starting with non-manipulation and following it with *nm*. The additional boys that are born as a result of this switch is enough to reverse the decrease in gender ratio found in Example 2 and make the equilibrium gender imbalance even higher than under the one-child policy.

As noted above, the couples in the intersection of $A_n(r^*)$ and $A_{mnm}(r^{**})$ will not manipulate if they are restricted to having only one child because their daughter-value index is sufficiently high. Moreover, as evidenced by their willingness to manipulate in the second stage if and only if they did not get a son the first time, they prefer mixed-gender children. Despite this, however, they will manipulate their first birth if they know that they can have two children. This is driven by our assumption that $\delta_{G|G} = 0.5 < 0.8 = \delta_{B|B}$, which, all other things being equal, makes the value of a second daughter only about 63% of the value of a second son. Since manipulation is not guaranteed to yield a son, this is enough to make the couples want to manipulate their first birth to reduce the possibility of ending up with two daughters. Although this also increases the probability of having two sons, they find this risk more palatable than the risk of having two daughters.

4 Conclusion

This paper builds a model of parental decision making in which parents decide whether to manipulate the birth process to increase the likelihood of obtaining a son and investigates its equilibrium under the one-child and the two-child policy settings. In addition, starting from an example that has been calibrated to match the current gender ratio in China, we provide parameter values for which the gender ratio initially improves under the two-child policy as parents who gave birth under the one-child policy make their manipulation decisions for their second child. We then show that, for the same parameters, the gender ratio worsens when a new set of parents make decisions for both their first and their second children. These results suggest that, as long as the underlying preference for sons remains intact, gender imbalance may not improve even when China fully implements a two-child policy. Furthermore, even if there are some initial improvements, they may not translate into a long-term improvement.

Although this paper is not arguing that the gender imbalance will necessarily worsen under a two-child policy, its results do suggest that the policy makers should not rely solely on relaxing the fertility limit to improve the gender ratio. Indeed, our examples demonstrate that, instead of automatically moderating the temptation to manipulate, the possibility of having a second child opens up additional channels through which incidence of manipulation may increase. In particular, if the value of a second daughter is sufficiently lower than the value of a second son, parents who would not have manipulated under the one-child policy may preemptively manipulate their first parity under the two-child policy to avoid the possibility of ending up with two daughters. Moreover, our example shows that this can happen even when parents have a preference for children of mixed genders. Therefore, policies that raise the value of a second daughter may be particularly important in improving the gender imbalance under a two-child policy. For example, in addition to the policies that have been advocated to raise the relative value of daughters in general, such as social security system to make parents less dependent on sons for old-age support (Banister [2]; Das Gupta et al. [6]), a financial subsidy to parents with two daughters may be considered.

References

[1] Fred Arnold and Zhaoxiang Liu. Sex preference, fertility, and family planning in China. *Population and Development Review*, 12:221–246, 1986.

[2] Judith Banister. Shortage of girls in China today. *Journal of Population Research*, 21:19–45, 2004.

[3] V. Bhaskar. Sex selection and gender balance. *American Economic Journal: Microeconomics*, 3:214–244, 2011.

[4] Heeran Chun and Monica Das Gupta. Gender discrimination in sex selective abortions and its transition in South Korea. *Women's Studies International Forum*, 32:89–97, 2009.

[5] Monica Das Gupta. Explaining Asia's "missing women": A new look at the data. *Population and Development Review*, 31:529–535, 2005.

[6] Monica Das Gupta, Zhenghua Jiang, Bohua Li, Zhenming Xie, Woojin Chung, and Hwa-Ok Bae. Why is son preference so persistent in East and South Asia? a cross-country study of China, India and the Republic of Korea. *Journal of Development Studies*, 40:153–187, 2003.

[7] James B. Davies and Junsen Zhang. The effects of gender control on fertility and children's consumption. *Journal of Population Economics*, 10:67–85, 1997.

[8] Weili Ding and Yuan Zhang. When a son is born: The impact of fertility patterns on family finance in rural China. *China Economic Review*, 30:192–208, 2014.

[9] Avraham Ebenstein. The "missing girls" of China and the unintended consequences of the one child policy. *Journal of Human Resources*, 45:87–115, 2010.

[10] Avraham Ebenstein. Estimating a dynamic model of sex selection in China. *Demography*, 48:783–811, 2011.

[11] Christophe Z. Guilmoto. The sex ratio transition in Asia. *Population and Development Review*, 35:519–549, 2009.

[12] Susan Harlap. Gender of infants conceived on different days of the menstrual cycle. *New England Journal of Medicine*, 300:1445–1448, 1979.

[13] Jinyoung Kim. Sex selection and fertility in a dynamic model of conception and abortion. *Journal of Population Economics*, 18:41–67, 2005.

[14] Hongbin Li, Junjian Yi, and Junsen Zhang. Estimating the effect of the one-child policy on the sex ratio imbalance in China: Identification based on the difference-in-differences. *Demography*, 48:1535–1557, 2011.

[15] Chai Bin Park and Nam-Hoon Cho. Consequences of son preference in a low-fertility society: Imbalance of the sex ratio at birth in Korea. *Population and Development Review*, 21:59–84, 1995.

[16] Hesketh Therese and Zhu Wei Xing. Abnormal sex ratios in human populations: Causes and consequences. *Proceedings of the National Academy of Sciences of the United States of America*, pages 13271–13275, 2006.

[17] Shang-Jin Wei and Xiaobo Zhang. The competitive saving motive: Evidence from rising sex ratios and savings in China. *Journal of Political Economy*, 119:511–564, 2011.

[18] Allen J. Wilcox, Clarice R. Weinberg, and Donna D. Baird. Timing of sexual intercourse in relation to ovulation – effects on the probability of conception, survival of the pregnancy, and sex of the baby. *New England Journal of Medicine*, 333:1517–1521, 1995.

[19] Yi Zeng, Ping Tu, Baochang Gu, Yi Xu, Bohua Li, and Yongpiing Li. Causes and implications of the recent increase in the reported sex ratio at birth in China. *Population and Development Review*, 19:283–302, 1993.

[20] Junsen Zhang, Jie Zhang, and Tianyou Li. Gender bias and economic development in an endogenous growth model. *Journal of Development Economics*, 59:497–525, 1999.

A Appendix

Table 1: Probability of having children of genders $g_1 g_2$ when taking action $a_1 a_B a_G$.

$a_1 a_B a_G$	BB	BG	GB	GG
nnn	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$
mnn	$\frac{p_B}{2}$	$\frac{p_B}{2}$	$\frac{p_G}{2}$	$\frac{p_G}{2}$
nnm	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{p_B}{2}$	$\frac{p_G}{2}$
mnm	$\frac{p_B}{2}$	$\frac{p_B}{2}$	$p_G p_B$	$p_G p_G$
nmm	$\frac{p_B}{2}$	$\frac{p_G}{2}$	$\frac{p_B}{2}$	$\frac{p_G}{2}$
mmm	$p_B p_B$	$p_B p_G$	$p_G p_B$	$p_G p_G$

Lemma 1. *We have*

$$u_m^i > u_n^i \iff V_d^i < \frac{r_G^i}{r_B^i}.$$

Proof. Suppose $r_G^i \leq r_B^i$. Then simplifying inequality (1) yields

$$\begin{aligned}
u_m^i > u_n^i \iff u_B^i > u_G^i &\iff M_B^i \pi_B^i + S_B^i(1-\pi_B^i) > M_G^i \pi_G^i + S_G^i(1-\pi_G^i) \\
&\iff M_B^i \left(\frac{r_G^i}{r_B^i}\right)\theta + S_B^i\left(1-\left(\frac{r_G^i}{r_B^i}\right)\theta\right) > M_G^i\theta + S_G^i(1-\theta) \\
&\iff \left(\theta M_B^i - \theta S_B^i\right)\left(\frac{r_G^i}{r_B^i}\right) > \theta M_G^i + (1-\theta)S_G^i - S_B^i \\
&\iff \frac{\left(\theta M_G^i + (1-\theta)S_G^i\right) - S_B^i}{\left(\theta M_B^i + (1-\theta)S_B^i\right) - S_B^i} < \frac{r_G^i}{r_B^i}. \qquad (6)
\end{aligned}$$

When $r_G^i > r_B^i$, we obtain

$$\begin{aligned}
u_m^i > u_n^i \iff u_B^i > u_G^i &\iff M_B^i \pi_B^i + S_B^i(1-\pi_B^i) > M_G^i \pi_G^i + S_G^i(1-\pi_G^i) \\
&\iff M_B^i\theta + S_B^i(1-\theta) > M_G^i\left(\frac{r_B^i}{r_G^i}\right)\theta + S_G^i\left(1-\left(\frac{r_B^i}{r_G^i}\right)\theta\right) \\
&\iff M_B^i\theta + S_B^i(1-\theta) > M_G^i\left(\theta - \theta + \left(\frac{r_B^i}{r_G^i}\right)\theta\right) + S_G^i\left(1-\theta+\theta-\left(\frac{r_B^i}{r_G^i}\right)\theta\right) \\
&\iff \left(M_B^i - M_G^i\right)\theta + \left(S_B^i - S_G^i\right)(1-\theta) > \left(S_G^i - M_G^i\right)\left(\theta - \left(\frac{r_B^i}{r_G^i}\right)\theta\right).
\end{aligned}$$

Under our assumption that $M_B^i \geq M_G^i > S_B^i \geq S_G^i$, the left side of the last inequality is greater than or equal to zero, while the right side is negative. Thus, u_m^i is always greater than u_n^i. Moreover, since the daughter-value index is always less than or equal to one, inequality (6) is also always satisfied when $r_G^i > r_B^i$. Thus, we trivially have

$$u_m^i > u_n^i \iff \frac{\left(\theta M_G^i + (1-\theta)S_G^i\right) - S_B^i}{\left(\theta M_B^i + (1-\theta)S_B^i\right) - S_B^i} < \frac{r_G^i}{r_B^i}.$$

$\square$

Theorem 2. *Suppose $f(S_B, M_G) > 0$ for all $(S_B, M_G) \in \Omega$. Then a unique equilibrium $r^* = (r_B^*, r_G^*)$ exists for each $p_B \in (\frac{1}{2}, 1)$ and $\theta \in (0,1]$. Moreover, the equilibrium gender ratio is always greater than one and is increasing in p_B and decreasing in θ.*

Proof. To make clear the dependence of A_m, μ_m, and ρ_B on the parameters of the model, as well as the gender ratio, we explicitly include them in the notation now. Thus, we use

$$\begin{aligned} A_m(r,\theta) &= \left\{(S_B, M_G) \in \Omega : \; M_G \; < \; \frac{r_G}{r_B} + \left(\frac{1}{\theta} - \frac{r_G}{r_B}\right) S_B\right\}, \\ \mu_m(r,\theta) &= \int_{A_m(r,\theta)} f(S_B, M_G)\, d(S_B, M_G), \\ \text{and} \quad \rho_B(r, p_B, \theta) &= \frac{1}{2} + \left(p_B - \frac{1}{2}\right)\left(\mu_m(r,\theta)\right). \end{aligned}$$

Fix any $p_B \in (\frac{1}{2}, 1)$ and $\theta \in (0,1]$. Since $\mu_m(\cdot,\theta)$ is a continuous function of r_B on $[\frac{1}{2}, 1]$, so is $\rho_B(\cdot, p_B, \theta)$. Moreover, $p_B \in (\frac{1}{2}, 1)$ and $\mu_m(r,\theta) \in [0,1]$ means $\rho_B(r, p_B, \theta) \in [\frac{1}{2}, 1]$. Therefore, $\rho_B(\cdot, p_B, \theta)$ is a continuous function of r_B from $[\frac{1}{2}, 1]$ into itself. Since $[\frac{1}{2}, 1]$ is nonempty, compact, and convex, Brouwer's fixed point theorem implies that there exists $r_B^* \in [\frac{1}{2}, 1]$ satisfying $r_B^* = \rho_B(r^*, p_B, \theta)$. This shows that an equilibrium always exists.

Next, we have

$$\begin{aligned} \frac{\partial}{\partial r_B}\left[\frac{r_G}{r_B} + \left(\frac{1}{\theta} - \frac{r_G}{r_B}\right) S_B\right] &= \frac{\partial}{\partial r_B}\left[\frac{S_B}{\theta} + (1 - r_B) r_B^{-1}(1 - S_B)\right] \\ &= \left(-r_B^{-1} - (1 - r_B) r_B^{-2}\right)(1 - S_B) \\ &= \begin{cases} (-) & \text{if } S_B < 1 \\ 0 & \text{if } S_B = 1. \end{cases} \end{aligned}$$

Thus, $A_m(r'', \theta) \subsetneq A_m(r', \theta)$ if $r_B'' > r_B'$. Since $f > 0$ on Ω, this implies that $\mu_m(r,\theta)$ and $\rho_B(r, p_B, \theta)$ are decreasing in r_B. Suppose that we have two equilibria, r^* and $\hat{r}$. We can assume without loss of generality that $r_B^* > \hat{r}_B$, but then we have

$$r_B^* \; = \; \rho_B(r^*, p_B, \theta) \; < \; \rho_B(\hat{r}, p_B, \theta) \; = \; \hat{r}_B,$$

which is a contradiction. Therefore, the equilibrium must be unique.

As noted in the main text, the equilibrium gender ratio cannot be less than one. To see that it cannot equal one, suppose $r_B^* = \frac{1}{2}$. Then $r_G^*/r_B^* = 1$ and $1/\theta - r_G^*/r_B^* \geq 0$, so $A_m(r^*, \theta) = \Omega$. This in turn means

$$\frac{1}{2} \; = \; r_B^* \; = \; \rho_B(r^*, p_B, \theta) \; = \; \frac{1}{2} + \left(p_B - \frac{1}{2}\right)\left(\mu_m(r^*, \theta)\right) \; = \; p_B \; > \; \frac{1}{2},$$

which is a contradiction.

For comparative statics, let $p''_B > p'_B$, and let r'' and r' be their corresponding equilibria. Suppose $r''_B \le r'_B$. Since the equilibrium gender ratio is always greater than one, $\mu_m(r',\theta) \neq 0$. Thus, we have

$$r'_B \;=\; \rho_B(r', p'_B, \theta) \;<\; \rho_B(r', p''_B, \theta) \;\le\; \rho_B(r'', p''_B, \theta) \;=\; r''_B,$$

which is a contradiction. Thus, $r''_B > r'_B$, which means that the equilibrium gender ratio is increasing in p_B. Finally, let $\theta'' > \theta'$, and let r'' and r' be their corresponding equilibria. Suppose $r''_B \ge r'_B$. Then, since $\rho_B(r, p_B, \theta)$ is decreasing in θ, we have

$$r'_B \;=\; \rho_B(r', p_B, \theta') \;\ge\; \rho_B(r'', p_B, \theta') \;>\; \rho_B(r'', p_B, \theta'') \;=\; r''_B,$$

which is a contradiction. Thus, the equilibrium gender ratio is decreasing in θ. $\square$

Lemma 3. *Given belief r^i, couple i's optimal strategy in the second stage takes the form $a_B a_G$, where a_{g_1} is the action choice if their first child is of gender g_1. Moreover, it is given by*

$$a_B a_G \;=\; \begin{cases} mm & \text{if } V^i_{d|B} \;<\; \frac{r^i_G}{r^i_B} \\ nm & \text{if } V^i_{d|G} \;<\; \frac{r^i_G}{r^i_B} \;\le\; V^i_{d|B} \\ nn & \text{if } V^i_{d|G} \;\ge\; \frac{r^i_G}{r^i_B}. \end{cases}$$

Proof. We have

$$\begin{aligned} u^i_{m|g_1} \;>\; u^i_{n|g_1} &\iff p_B\left(\delta_{B|g_1} u^i_B\right) + (1-p_B)\left(\delta_{G|g_1} u^i_G\right) \;>\; \tfrac{1}{2}\left(\delta_{B|g_1} u^i_B\right) + \tfrac{1}{2}\left(\delta_{G|g_1} u^i_G\right) \\ &\iff \left(p_B - \tfrac{1}{2}\right)\delta_{B|g_1} u^i_B \;>\; \left(p_B - \tfrac{1}{2}\right)\delta_{G|g_1} u^i_G \\ &\iff u^i_B \;>\; \frac{\delta_{G|g_1}}{\delta_{B|g_1}} u^i_G. \end{aligned} \tag{7}$$

Suppose $u^i_{m|B} > u^i_{n|B}$. Since inequality (7) implies

$$u^i_B \;>\; \frac{\delta_{G|B}}{\delta_{B|B}} u^i_G \;>\; u^i_G \;\ge\; \frac{\delta_{G|G}}{\delta_{B|G}} u^i_G,$$

we also have $u^i_{m|G} > u^i_{n|G}$. Thus, for any $a_B a_G \neq mm$,

$$u^i_{a_1 mm} \;=\; u^i_{a_1} + p_{B|a_1} u^i_{m|B} + p_{G|a_1} u^i_{m|G} \;>\; u^i_{a_1} + p_{B|a_1} u^i_{a_B|B} + p_{G|a_1} u^i_{a_G|G} \;=\; u^i_{a_1 a_B a_G}.$$

Therefore, the optimal strategy in this case must be mmm or nmm.

Next, suppose $u^i_{m|G} \le u^i_{n|G}$. Inequality (7) now yields

$$u^i_B \;\le\; \frac{\delta_{G|G}}{\delta_{B|G}} u^i_G \;<\; u^i_G \;<\; \frac{\delta_{G|B}}{\delta_{B|B}} u^i_G,$$

Thus, we also have $u^i_{m|B} < u^i_{n|B}$. This implies that for any $a_B a_G \neq nn$,

$$u^i_{a_1 nn} \;=\; u^i_{a_1} + p_{B|a_1} u^i_{n|B} + p_{G|a_1} u^i_{n|G} \;\ge\; u^i_{a_1} + p_{B|a_1} u^i_{a_B|B} + p_{G|a_1} u^i_{a_G|G} \;=\; u^i_{a_1 a_B a_G}.$$

Since we have assumed the couples choose n when they are indifferent between n and m, the optimal strategy in this case must be mnn or nnn.

Lastly, suppose $u^i_{m|B} \le u^i_{n|B}$ and $u^i_{m|G} > u^i_{n|G}$. Then

$$u^i_{a_1nm} = u^i_{a_1} + p_{B|a_1}u^i_{n|B} + p_{G|a_1}u^i_{m|G} \ge u^i_{a_1} + p_{B|a_1}u^i_{m|B} + p_{G|a_1}u^i_{m|G} = u^i_{a_1mm},$$

and for any $a_Ba_G \notin \{nm, mm\}$, we have

$$u^i_{a_1nm} = u^i_{a_1} + p_{B|a_1}u^i_{n|B} + p_{G|a_1}u^i_{m|G} > u^i_{a_1} + p_{B|a_1}u^i_{a_B|B} + p_{G|a_1}u^i_{a_G|G} = u^i_{a_1a_Ba_G}.$$

Therefore, the only possible optimal strategy is mnm or nnm.

Putting all the cases together shows that the optimal second-stage strategy is given by:

$$a_Ba_G = \begin{cases} mm & \text{if } u^i_{m|B} > u^i_{n|B} \\ nm & \text{if } u^i_{m|B} \le u^i_{n|B} \text{ and } u^i_{m|G} > u^i_{n|G} \\ nn & \text{if } u^i_{m|G} \le u^i_{n|G}. \end{cases}$$

Further simplifying inequality (7) yields

$$\begin{aligned} u^i_{m|g_1} > u^i_{n|g_1} &\iff \delta_{G|g_1}u^i_G < \delta_{B|g_1}u^i_B \\ &\iff \delta_{G|g_1}\left(M^i_G\theta + S^i_G(1-\theta)\right) < \delta_{B|g_1}\left(M^i_B\theta\left(\frac{r^i_G}{r^i_B}\right) + S^i_B\left[1-\theta\left(\frac{r^i_G}{r^i_B}\right)\right]\right) \\ &\iff \frac{\delta_{G|g_1}\left(M^i_G\theta + S^i_G(1-\theta)\right) - \delta_{B|g_1}S^i_B}{\delta_{B|g_1}\left(M^i_B\theta - S^i_B\theta\right)} < \frac{r^i_G}{r^i_B} \\ &\iff \frac{\delta_{G|g_1}\left(M^i_G\theta + S^i_G(1-\theta)\right) - \delta_{B|g_1}S^i_B}{\delta_{B|g_1}\left(M^i_B\theta + S^i_B(1-\theta)\right) - \delta_{B|g_1}S^i_B} < \frac{r^i_G}{r^i_B}. \end{aligned}$$

Therefore, the optimal second stage strategy can be expressed as:

$$a_Ba_G = \begin{cases} mm & \text{if } V^i_{d|B} < \frac{r^i_G}{r^i_B} \\ nm & \text{if } V^i_{d|G} < \frac{r^i_G}{r^i_B} \le V^i_{d|B} \\ nn & \text{if } V^i_{d|G} \ge \frac{r^i_G}{r^i_B}. \end{cases}$$

□

Theorem 4. *Suppose that the two-child policy is announced after the couples had their first child, and the couples react to the policy without changing their belief about the gender ratio. Then the expected gender ratio decreases relative to the one child-policy if and only if* $p_B\mu_{mnm}(r^*) > \frac{1}{2}\mu_{nnm}(r^*)$.

Proof. Using equation (4) and the fact that $\mu_{a_1}(r^*) = \mu_{a_1a_1a_1}(r^*) + \mu_{a_1nm}(r^*)$ yields

$$
\begin{aligned}
\rho_{B_2}\left(r^*\right) < \rho_B\left(r^*\right) \iff\; & p_B\mu_{mmm}\left(r^*\right) + \left(\tfrac{p_B}{2} + p_B(1-p_B)\right)\mu_{mnm}\left(r^*\right) \\
& \quad + \left(\tfrac{1}{2}\right)\mu_{nnn}\left(r^*\right) + \left(\tfrac{1}{4} + \tfrac{p_B}{2}\right)\mu_{nnm}\left(r^*\right) \;<\; p_B\mu_m\left(r^*\right) + \tfrac{1}{2}\mu_n\left(r^*\right) \\
\iff\; & \left(\tfrac{1}{4} + \tfrac{p_B}{2} - \tfrac{1}{2}\right)\mu_{nnm}\left(r^*\right) \;<\; \left(p_B - \left(\tfrac{p_B}{2} + p_B(1-p_B)\right)\right)\mu_{mnm}\left(r^*\right) \\
\iff\; & \tfrac{1}{2}\left(p_B - \tfrac{1}{2}\right)\mu_{nnm}\left(r^*\right) \;<\; p_B\left(p_B - \tfrac{1}{2}\right)\mu_{mnm}\left(r^*\right) \\
\iff\; & \tfrac{1}{2}\mu_{nnm}\left(r^*\right) \;<\; p_B\mu_{mnm}\left(r^*\right).
\end{aligned}
$$

□

Lemma 5. *Suppose that, given belief r^i, a_Ba_G is the optimal second-stage strategy for couple i. Then ma_Ba_G is the overall optimal strategy if*

$$
M_G^i \;<\; \frac{1 + p_{B|a_B}\delta_{B|B} - p_{B|a_G}\delta_{B|G}}{1 + p_{G|a_G}\delta_{G|G} - p_{G|a_B}\delta_{G|B}}\left[\frac{r_G^i}{r_B^i} + \left(\frac{1}{\theta} - \frac{r_G^i}{r_B^i}\right)S_B^i\right].
$$

Otherwise, na_Ba_G is optimal. Moreover, mnn is never optimal if $\delta_{B|G} = 1$, and nmm is never optimal if $\delta_{G|B} = 1$.

Proof. Simplifying inequality (5) yields

$$
\begin{aligned}
& u^i_{ma_Ba_G} \;>\; u^i_{na_Ba_G} \\
\iff\; & u_G^i + u^i_{a_G|G} \;<\; u_B^i + u^i_{a_B|B} \\
\iff\; & u_G^i + p_{B|a_G}\delta_{B|G}u_B^i + p_{G|a_G}\delta_{G|G}u_G^i \;<\; u_B^i + p_{B|a_B}\delta_{B|B}u_B^i + p_{G|a_B}\delta_{G|B}u_G^i \\
\iff\; & u_G^i\left(1 + p_{G|a_G}\delta_{G|G} - p_{G|a_B}\delta_{G|B}\right) \;<\; u_B^i\left(1 + p_{B|a_B}\delta_{B|B} - p_{B|a_G}\delta_{B|G}\right) \\
\iff\; & M_G^i\theta + S_G^i(1-\theta) \;<\; \frac{1 + p_{B|a_B}\delta_{B|B} - p_{B|a_G}\delta_{B|G}}{1 + p_{G|a_G}\delta_{G|G} - p_{G|a_B}\delta_{G|B}}\left(M_B^i\left(\frac{r_G^i}{r_B^i}\right)\theta + S_B^i\left[1 - \left(\frac{r_G^i}{r_B^i}\right)\theta\right]\right) \\
\iff\; & M_G^i \;<\; \frac{1 + p_{B|a_B}\delta_{B|B} - p_{B|a_G}\delta_{B|G}}{1 + p_{G|a_G}\delta_{G|G} - p_{G|a_B}\delta_{G|B}}\left[\frac{r_G^i}{r_B^i} + \left(\frac{1}{\theta} - \frac{r_G^i}{r_B^i}\right)S_B^i\right].
\end{aligned}
$$

Now, let $\delta_{B|G} = 1$. Then

$$
\begin{aligned}
\frac{1 + \frac{1}{2}\delta_{B|B} - \frac{1}{2}\delta_{B|G}}{1 + \frac{1}{2}\delta_{G|G} - \frac{1}{2}\delta_{G|B}} \;<\; \frac{\delta_{B|G}}{\delta_{G|G}} \iff\; & \frac{1 + \frac{1}{2}\delta_{B|B} - \frac{1}{2}}{1 + \frac{1}{2}\delta_{G|G} - \frac{1}{2}\delta_{G|B}} \;<\; \frac{1}{\delta_{G|G}} \\
\iff\; & \delta_{G|G} + \tfrac{1}{2}\delta_{B|B}\delta_{G|G} - \tfrac{1}{2}\delta_{G|G} \;<\; 1 + \tfrac{1}{2}\delta_{G|G} - \tfrac{1}{2}\delta_{G|B} \\
\iff\; & \delta_{G|G} + \tfrac{1}{2}\delta_{B|B}\delta_{G|G} + \tfrac{1}{2}\delta_{G|B} \;<\; 1 + \delta_{G|G} \\
\iff\; & \tfrac{1}{2}\delta_{B|B}\delta_{G|G} + \tfrac{1}{2}\delta_{G|B} \;<\; 1.
\end{aligned}
$$

The last inequality always holds since $\delta_{B|B}\delta_{G|G} < 1$ and $\delta_{G|B} \leq 1$. The first inequality implies that nnn is optimal whenever nn is optimal. Therefore, mnn is never optimal.

Next, let $\delta_{G|B} = 1$. Then

$$\begin{aligned}
& \frac{1+p_B\delta_{B|B}-p_B\delta_{B|G}}{1+(1-p_B)\delta_{G|G}-(1-p_B)\delta_{G|B}} \;>\; \frac{\delta_{B|B}}{\delta_{G|B}} \\
\iff\; & \frac{1+p_B\delta_{B|B}-p_B}{1+(1-p_B)\delta_{G|G}-(1-p_B)} \;>\; \delta_{B|B} \\
\iff\; & 1+p_B\delta_{B|B}-p_B \;>\; \delta_{B|B}+(1-p_B)\delta_{B|B}\delta_{G|G}-(1-p_B)\delta_{B|B} \\
\iff\; & 1-p_B \;>\; (1-p_B)\delta_{B|B}\delta_{G|G}.
\end{aligned}$$

The last inequality always holds since $\delta_{B|B}\delta_{G|G} < 1$. The first inequality implies that the mmm is optimal whenever mm is optimal. Therefore, nmm is never optimal. □

Theorem 6. *Suppose $f(S_B, M_G) > 0$ for all $(S_B, M_G) \in \Omega$ and $\delta_{B|G} = \delta_{G|B} = 1$. Then a unique equilibrium $r^{**} = \left(r_B^{**}, r_G^{**}\right)$ exists for each $p_B \in \left(\frac{1}{2}, 1\right)$, $\theta \in (0,1]$, and discount factors $1 > \delta_{B|B} \geq \delta_{G|G} > 0$. Moreover, the equilibrium gender ratio is increasing in $\delta_{B|B}$ and decreasing in $\delta_{G|G}$.*

Proof. As in the proof of Theorem 2, we explicitly include the relevant parameters, p_B, θ, and $\delta = (\delta_{B|G}, \delta_{G|B}, \delta_{B|B}, \delta_{G|G})$, in our notation below. By Lemma 5, $A_{nmm}(r,\theta,\delta)$ and $A_{mnn}(r,\theta,\delta) = \emptyset$ are empty. Thus,

$$\begin{aligned}
A_{mmm}(r,\theta,\delta) &= \left\{(S_B, M_G) \in \Omega : \; M_G \;<\; \frac{\delta_{B|B}}{\delta_{G|B}}\left[\frac{r_G}{r_B} + \left(\frac{1}{\theta} - \frac{r_G}{r_B}\right)S_B\right]\right\}, \\
A_{nm}(r,\theta,\delta) &= \left\{(S_B, M_G) \in \Omega : \; \frac{\delta_{B|B}}{\delta_{G|B}}\left[\frac{r_G}{r_B} + \left(\frac{1}{\theta} - \frac{r_G}{r_B}\right)S_B\right] \;\leq\; M_G \right. \\
&\qquad\qquad \left. <\; \frac{\delta_{B|G}}{\delta_{G|G}}\left[\frac{r_G}{r_B} + \left(\frac{1}{\theta} - \frac{r_G}{r_B}\right)S_B\right]\right\}, \\
A_{mnm}(r,\theta,\delta) &= \left\{(S_B, M_G) \in A_{nm}(r,\theta,\delta) : \right. \\
&\qquad\qquad \left. M_G \;<\; \frac{1+\frac{1}{2}\delta_{B|B}-p_B\delta_{B|G}}{1+(1-p_B)\delta_{G|G}-\frac{1}{2}\delta_{G|B}}\left[\frac{r_G}{r_B} + \left(\frac{1}{\theta} - \frac{r_G}{r_B}\right)S_B\right]\right\}, \\
A_{nnm}(r,\theta,\delta) &= A_{nm}(r,\theta,\delta) \setminus A_{mnm}(r,\theta,\delta), \text{ and} \\
A_{nnn}(r,\theta,\delta) &= \left\{(S_B, M_G) \in \Omega : \; M_G \;\geq\; \frac{\delta_{B|G}}{\delta_{G|G}}\left[\frac{r_G}{r_B} + \left(\frac{1}{\theta} - \frac{r_G}{r_B}\right)S_B\right]\right\}.
\end{aligned}$$

Let $\mathrm{E}[\#B|a_1 a_B a_G]$ be the expected number of boys that are produced by strategy $a_1 a_B a_G$. The induced proportion of boys reduces to

$$\begin{aligned}
\rho_B(r, p_B, \theta, \delta) &= \tfrac{1}{2}\Big(\mathrm{E}[\#B|mmm]\,\mu_{mmm}(r,\theta,\delta) + \mathrm{E}[\#B|mnm]\,\mu_{mnm}(r,\theta,\delta) \\
&\qquad + \mathrm{E}[\#B|nnm]\,\mu_{nnm}(r,\theta,\delta) + \mathrm{E}[\#B|nnn]\,\mu_{nnn}(r,\theta,\delta)\Big) \\
&= \tfrac{1}{2}\left((2p_B)\,\mu_{mmm}(r,\theta,\delta) + \left(\tfrac{3p_B+2p_Bp_G}{2}\right)\mu_{mnm}(r,\theta,\delta)\right. \\
&\qquad \left. + \left(\tfrac{3+2p_B}{4}\right)\mu_{nnm}(r,\theta,\delta) + \mu_{nnn}(r,\theta,\delta)\right).
\end{aligned}$$

Fix any $p_B \in \left(\frac{1}{2}, 1\right)$, $\theta \in (0,1]$, and δ satisfying $\delta_{B|G} = \delta_{G|B} = 1 > \delta_{B|B} \geq \delta_{G|G} > 0$. For all $a_1 a_B a_G$, $\mu_{a_1 a_B a_G}(\cdot, \theta, \delta)$ is a continuous function of r_B on $\left[\frac{1}{2}, 1\right]$. As in the proof of

Theorem 2, this implies that $\rho_B(\cdot, p_B, \theta, \delta)$ is a continuous function of r_B from $\left[\frac{1}{2}, 1\right]$ into itself, and Brouwer's fixed point theorem implies that there exists $r_B^{**} \in \left[\frac{1}{2}, 1\right]$ satisfying $r_B^{**} = \rho_B(r^{**}, p_B, \theta)$. This shows that an equilibrium always exists.

Let $r'_B > r_B$, and let

$$\Delta\rho_B\left(a_1 a_B a_G(r) \to a'_1 a'_B a'_G(r')\right) = \left(\mathrm{E}[\#B|a'_1 a'_B a'_G] - \mathrm{E}[\#B|a_1 a_B a_G]\right) \times \tilde{\mu}\left(A_{a_1 a_B a_G}(r,\theta,\delta) \cap A_{a'_1 a'_B a'_G}(r',\theta,\delta)\right)$$

be the change in the expected number of boys that are produced by couples who switch from using $a_1 a_B a_G$ to $a'_1 a'_B a'_G$ when the belief changes from r_B to r'_B, weighted by the fraction of couples who make the switch. As we have shown in the proof of Theorem 2,

$$\frac{\partial}{\partial r_B}\left[\frac{r_G}{r_B} + \left(\frac{1}{\theta} - \frac{r_G}{r_B}\right) S_B\right] = \begin{cases} (-) & \text{if } S_B < 1 \\ 0 & \text{if } S_B = 1. \end{cases}$$

Since $r'_B > r_B$, this implies that $A_{mmm}(r',\theta,\delta) \subsetneq A_{mmm}(r,\theta,\delta)$ and $A_{nnn}(r',\theta,\delta) \supseteq A_{nnn}(r,\theta,\delta)$. In addition, the boundary between $A_{mnm}(r',\theta,\delta)$ and $A_{nnm}(r',\theta,\delta)$ is lower than the boundary between $A_{mnm}(r,\theta,\delta)$ and $A_{nnm}(r,\theta,\delta)$. Thus,

$$\begin{aligned}\rho_B(r', p_B, \theta, \delta) - \rho_B(r, p_B, \theta, \delta) = {} & \Delta\rho_B\left(mmm(r) \to mnm(r')\right) + \Delta\rho_B\left(mmm(r) \to nnm(r')\right) \\ & + \Delta\rho_B\left(mmm(r) \to nnn(r')\right) + \Delta\rho_B\left(mnm(r) \to nnm(r')\right) \\ & + \Delta\rho_B\left(mnm(r) \to nnn(r')\right) + \Delta\rho_B\left(nnm(r) \to nnn(r')\right).\end{aligned}$$

Since $p_B > \frac{1}{2}$, we have

$$\mathrm{E}[\#B|mmm] > \mathrm{E}[\#B|mnm] > \mathrm{E}[\#B|nnm] > \mathrm{E}[\#B|nnn].$$

This, combined with the fact that $\tilde{\mu}\left(A_{mmm}(r,\theta,\delta) \cap A_{mnm}(r',\theta,\delta)\right) > 0$, yields

$$\rho_B(r', p_B, \theta, \delta) - \rho_B(r, p_B, \theta, \delta) < 0.$$

Therefore, $\rho_B(r, p_B, \theta, \delta)$ is decreasing in r_B. Reasoning similar to the proof of Theorem 2 then implies that the equilibrium is unique.

Suppose $\delta'_{B|B} > \delta_{B|B}$, $\delta = (\delta_{B|G}, \delta_{G|B}, \delta_{B|B}, \delta_{G|G})$, and $\delta' = (\delta_{B|G}, \delta_{G|B}, \delta'_{B|B}, \delta_{G|G})$. Then $A_{nnn}(r,\theta,\delta') = A_{nnn}(r,\theta,\delta)$, $A_{mmm}(r,\theta,\delta') \supsetneq A_{mmm}(r,\theta,\delta)$, and $A_{nnm}(r,\theta,\delta') \subsetneq A_{nnm}(r,\theta,\delta)$. Letting

$$\Delta\rho_B\left(a_1 a_B a_G(\delta) \to a'_1 a'_B a'_G(\delta')\right) = \left(\mathrm{E}[\#B|a'_1 a'_B a'_G] - \mathrm{E}[\#B|a_1 a_B a_G]\right) \times \tilde{\mu}\left(A_{a_1 a_B a_G}(r,\theta,\delta) \cap A_{a'_1 a'_B a'_G}(r,\theta,\delta')\right),$$

we have

$$\begin{aligned}\rho_B(r, p_B, \theta, \delta') - \rho_B(r, p_B, \theta, \delta) = {} & \Delta\rho_B\left(mnm(\delta) \to mmm(\delta')\right) + \Delta\rho_B\left(nnm(\delta) \to mmm(\delta')\right) \\ & + \Delta\rho_B\left(nnm(\delta) \to mnm(\delta')\right) \\ > {} & 0.\end{aligned}$$

Next, suppose $\delta'_{G|G} > \delta_{G|G}$, $\delta = (\delta_{B|G}, \delta_{G|B}, \delta_{B|B}, \delta_{G|G})$, and $\delta' = (\delta_{B|G}, \delta_{G|B}, \delta_{B|B}, \delta'_{G|G})$. Then $A_{mmm}(r,\theta,\delta') = A_{mmm}(r,\theta,\delta)$, $A_{nnn}(r,\theta,\delta') \supseteq A_{nnn}(r,\theta,\delta)$, and $A_{mnm}(r,\theta,\delta') \subsetneq A_{mnm}(r,\theta,\delta)$. Thus,

$$\begin{aligned}
\Delta\rho_B\left(a_1 a_B a_G(\delta) \to a'_1 a'_B a'_G(\delta')\right) &= \Delta\rho_B\left(nnm(\delta) \to nnn(\delta')\right) + \Delta\rho_B\left(mnm(\delta) \to nnn(\delta')\right) \\
&\quad + \Delta\rho_B\left(mnm(\delta) \to nnm(\delta')\right) \\
&< 0.
\end{aligned}$$

Therefore, $\rho_B(r, p_B, \theta, \delta)$ is increasing in $\delta_{B|B}$ and decreasing in $\delta_{G|G}$. Reasoning similar to the proof of Theorem 2 then implies that the equilibrium gender ratio is increasing in $\delta_{B|B}$ and decreasing in $\delta_{G|G}$. □