# Will the needs-based planning of health human resources currently undertaken in several countries lead to excess supply and inefficiency?*

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**Abstract**

Recently, the emphasis in Health Human Resources (HHR) planning has shifted away from a utilization-based approach towards a needs-based one in which planning is based on the projected health needs of the population. However, needs-based models that are currently in use rely on a definition of "needs" that include only the medical circumstances of individuals and not personal preferences or other socio-economic factors. We examine whether planning based on such a narrow definition will maximize social welfare. We show that, in a publicly funded healthcare system, if the planner seeks to meet the aggregate need without taking utilization into consideration, then over-supply of HHR is likely since "needs" do not necessarily translate into "usage." Our result suggests that HHR planning should track the healthcare system as access gradually improves since, even if healthcare is fully accessible, individuals may not fully utilize it to the degree prescribed by their medical circumstances.

Key words: Needs-based planning, Utilization-based planning, Social welfare, Health human resources.

JEL Classfications: I10, I12, I18.

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# 1 Introduction

The traditional, utilization-based Health Human Resources (HHR) planning methods forecast future HHR demand, or requirements, based on only the past rates of utilization for each demographic, such as age and gender, and the projected changes in these demographics. The main criticism of these methods has been that many other factors, such as technological innovations (new treatments or diagnostic services), epidemiological changes, personal preferences, and socio-economic factors, also affect the demand for healthcare providers. In view of this limitation, interest has recently shifted to needs-based approach in which planning is based on the projected health needs of a population.

For example, the World Health Organization (WHO) and the Pan American Health Organization (PAHO) established an international consortium on needs-based planning. The goal of the consortium, which currently includes Brazil, Canada, and Jamaica, is to develop the capacity of regional health authorities to base their HHR planning on "the needs of their populations and the productive capacity of their workforce" (WHO/PAHO Collaborating Centre, 2012). Several provinces in Canada, such as Nova Scotia and Ontario, are also relying on this principle for human resources planning for their physicians and nurses.

The call for needs-based planning has been particularly vocal in Canada. The Canadian Medical Association and the Canadian Nurses Association (2005) contend, "Planners need to adopt a needs-based approach that anticipates the current and emerging health needs of the population that are determined by demographic, epidemiological, cultural and geographic factors." Similarly, Task Force Two (2006) proposes that a pan-Canadian approach for HHR planning must include needs-based factors. The Advisory Committee on Health Delivery and Human Resources (2007) also advocates an approach that is driven by current and future population health needs rather than past utilization trends.

It is important to note, however, that the link between population health and its healthcare needs is not as simple and obvious as implicitly assumed by most advocates of needs-based planning since the linkage depends on numerous socio-economic factors. Grossman (1972a, 1972b) shows that the demand for medical care is determined by income, prices, the severity of the individual's illness, and the perceived efficacy of the treatment. Moreover, as recognized by Garber (2000), a patient who experiences a large disutility from a disease and only a small disutility from its treatment (for example, from side effects) may have a different preference than a patient with identical health characteristics who experiences only a small disutility from the disease but a large disutility from the treatment. Thus, patients with identical health characteristics can have different healthcare needs.

Because of the difficulty in determining the true healthcare needs of the population, studies on the needs-based planning have used a very narrow definition of "needs" that includes only the medical circumstances of the population.[1] For exam-

[1]Although we focus on the consequences of planning for needs that are not paired with utilization, the need for careful analysis of medical "needs" run deeper. A more general discussion of what should

ple, Birch et al. (2005) and Tomblin-Murphy et al. (2009) estimate the future requirements for the services of a given group of providers using only the size, distribution, and levels of medical needs of the population. Thus, despite being based on the conceptual framework of O'Brien-Pallas et al. (1992, 2001), which recognizes the social and economic factors in which HHR planning takes place, the socio-economic aspect of healthcare needs have been absent in these studies. Similarly, although Birch et al. (2007) discuss the importance of the socio-economic component in HHR planning, their actual model for estimating HHR requirements ignores these factors.

Thus, the needs-based models that are currently used in HHR planning are subject to the same criticism as the traditional models, except that they now rely on a very narrow definition of needs rather than utilization. In particular, because the "needs" are divorced from the influence of socio-economic factors and personal preferences, it is not clear whether "needs" will necessarily translate into "usages." It is therefore imperative that we study the possible consequences of the current needs-based approach, as healthcare systems of many countries and jurisdictions will be affected by such planning in the near future. In the following, we examine whether needs-based planning, in the context of a publicly funded healthcare system, will maximize social welfare if needs are decoupled from economic factors.

## 2 Model and Analysis

Our model of an economy with a publicly funded healthcare system features three goods: leisure, healthcare service, and the composite consumption good. Both healthcare service, represented by HHR, and the composite good are produced using a single production technology that uses labor as the only input. The central planner exogenously mandates the level of HHR that must be produced, while the market conditions dictate the production of the composite good. For each choice of HHR supply level, the economy functions like a standard general equilibrium economy, albeit with a constraint that the total healthcare usage cannot exceed the planner-set supply.

### 2.1 Private Components of the Economy

The productive capability of the economy is represented by a single firm with a twice-differentiable, strictly concave production technology $f(L)$, where $L$ is the labor input. The total output of the economy is divided between HHR and the composite good. We assume that the planner has first use of the productive capability, so the total composite good that can be produced is $f(L)-H^s$ if the planner sets the HHR supply level at $H^s$. Normalizing the price of HHR and the composite good to one and letting $w$ denote the wage rate, we let $L^d(w)$ be the firm's labor demand function and $\pi(w)$

constitute a "need" is given in Culyer (1998) and Hasman, Hope, and Østerdal (2006).

be its profit function.[2] Then $L^d(w)$ is strictly decreasing in $w$.[3]

There are $I$ individuals in the economy. Each individual $i$ derives utility from the consumption of the composite good ($c_i$), leisure ($\ell_i$), and HHR ($h_i$). The individual's utility is assumed to take the quasi-linear form:[4]

$$u_i(c_i,\ell_i,h_i) = c_i + \phi_i(\ell_i,h_i),$$

where $\phi_i$ is increasing in $\ell_i$ and $h_i$, $\frac{\partial^2\phi_i}{\partial\ell_i\partial h_i} > 0$, and $D^2\phi_i$ is negative definite. Negative definiteness implies the strict concavity of $\phi_i$. We interpret the requirement that the cross partials be positive as requiring that the marginal utility of leisure be increasing in HHR consumption, which seems plausible since higher healthcare consumption leads to better health, and that makes leisure more valuable. The cost of supplying HHR is shared by the individuals according to some fixed proportional rule, with $\theta_i H^s$ being $i$'s share.

A strand of research, in the vein of John Roemer's equality of opportunity theory (1998), has long recognized that an individual's health status depends not only on exogenous factors such as genetic makeup or physical environment but also on behavior (Fleurbaey and Schokkaert, 2009; Herrero and Moreno-Ternero, 2009; Rosa-Dias, 2009; Trannoy et al., 2010). Similarly, an individual's consumption of HHR in our model also depends on these two factors. Let $N_i \in \mathbb{R}_{++}$ be individual $i$'s healthcare need, which is determined exogenously by her medical circumstances, and let $e_i \in [0, \bar{e}_i]$ be the effort (measured in terms of time) spent on trying to obtain healthcare. We assume that individual $i$ exerting effort $e_i$ when her need is $N_i$ translates into a request for $g_i(e_i,N_i)$ units of HHR service, where $g_i$ is twice-differentiable, increasing, and (weakly) concave in $e_i$. We also assume that $g_i(0,N_i) = 0$ and $g_i(\bar{e}_i,N_i) = N_i$.

When the total demand for HHR exceeds the available supply, it must be rationed. In our model, the rationing rule is represented by a fraction $a(e,H^s) \in [0,1]$, which is determined endogenously by the vector of effort choices, $e = (e_1,...,e_I)$, and the supply of HHR. In particular, we assume that the actual level of HHR service the individual gets to consume is $h_i = a(e,H^s)g_i(e_i,N_i)$, where

$$a(e,H^s) = \min\left\{1, \frac{H^s}{\sum_i g_i(e_i,N_i)}\right\}.$$

[2]The prices of HHR and the composite good are assumed to be the same for simplicity. They can be allowed to be different without materially affecting the result. In addition, our specification assumes that the production process can freely convert one unit of HHR into one unit of the composite good and vice versa. The model can be modified so that HHR and the composite good are produced by separate production functions. However, in our view, this will complicate the model without adding new insight.

[3]The labor demand function $L^d(w)$ is a solution to $\max_L\ f(L) - wL$. Differentiating the first order condition $f'(L^d(w)) - w = 0$ with respect to $w$ yields

$$\frac{dL^d(w)}{dw} = \frac{1}{f''(L^d(w))} < 0.$$

[4]Deriving social welfare by aggregating individual welfare requires the individuals to have utility functions whose indirect utility takes the Gorman form. We further restrict attention to quasi-linear utility for tractability. See, for example, Chapter 4 of Mas-Colell, Whinston & Green (1995) for a discussion of the Gorman form and its aggregation properties.

We call $a(e,H^s)$ the accessibility level of the healthcare system.

**Remark.** Since $a(e,H^s)$ is the proportion of the aggregate healthcare need that society can meet, this rule gives every individual the same proportional access to the healthcare system. Arguably, such rule is normatively "fair" only if there are no other factors restricting the individual's access. Yet, Rosa Dias (2009) and Trannoy et al. (2010) show that individuals with different social backgrounds may have unequal opportunities to obtain healthcare service.

Our model recognizes that such inequality exists by allowing the function $g_i$, which translates effort and medical need into HHR demand, to be individual specific. For example, as in Fleurbaey and Schokkaert (2009), we can introduce a new parameter $s_i$ that indexes one's social standing, and let $g_i(e_i,N_i) = g(e_i,N_i,s_i)$, with $g(e,N,s_i) < g(e,N,s_j)$ if $s_i < s_j$. Then the resulting model captures the feature that an individual with lower social standing must exert greater effort to obtain the same level of healthcare as someone with the same medical condition but with higher social standing.

In such case, the society may want to compensate for this inequality by setting individual specific accessibility levels so that $a_i > a_j$. However, the precise way in which the compensation should occur depends necessarily on the objective of the society beyond mere efficiency, and lies outside the scope of this paper. Therefore, we abstract away from this issue and use the proportional rationing rule given above. □

Given an individual's accessibility level $a$ and her healthcare need, the amount of HHR she consumes is completely determined by her effort. Therefore, we can treat the individual's utility function as a function of $c_i$, $\ell_i$, and $e_i$. That is,

$$u_i(c_i,\ell_i,e_i;a,N_i) = c_i + \phi_i(\ell_i,e_i;a,N_i) = c_i + \phi_i(\ell_i,ag_i(e_i,N_i)).$$

**Example 1.** For a simple example of a utility function satisfying our requirements, let

$$u_i(c_i,\ell_i,h_i) = c_i + \ell_i^{\alpha_i} h_i^{\beta_i},$$

where $\alpha_i > 0$, $\beta_i > 0$, and $\alpha_i + \beta_i < 1$. Next, let $g_i(e_i,N_i) = \frac{N_i}{\bar{e}_i} e_i$. Then we have

$$u_i(c_i,\ell_i,e_i;a,N_i) = c_i + \ell_i^{\alpha_i}\left(\frac{aN_ie_i}{\bar{e}_i}\right)^{\beta_i},$$

Moreover, since

$$\frac{\partial^2 u_i}{\partial N_i \partial e_i} = \frac{\partial}{\partial e_i}\left[\beta_i \ell_i^{\alpha_i}\left(\frac{ae_i}{\bar{e}_i}\right)^{\beta_i} N_i^{\beta_i-1}\right] = \beta_i^2 \ell_i^{\alpha_i}\left(\frac{a}{\bar{e}_i}\right)^{\beta_i}(e_iN_i)^{\beta_i-1} > 0,$$

the marginal utility of effort is increasing in healthcare need for this utility function. □

Each individual in the economy faces a trade-off between time spent on earning income and those spent on leisure and healthcare. Let $\bar{L}_i$ denote individual $i$'s labor endowment, and let $\delta_i \pi(w)$ be $i$'s share of the firm's profit. The individual's optimization problem, given wage $w$, perceived accessibility level $a$, and healthcare burden $\theta_i H^s$, is:[5]

$$\max_{c_i,\ell_i,e_i} \; c_i + \phi_i(\ell_i, e_i; a, N_i) \quad \text{s.t.} \quad c_i \le \delta_i \pi(w) - \theta_i H^s + w(\bar{L}_i - \ell_i - e_i) \qquad (1)$$
$$\text{and} \quad e_i \le \bar{e}_i.$$

We call the requirement that $e_i \le \bar{e}_i$, the *effort constraint*.

Let $c_i(w,a,N_i)$, $\ell_i(w,a,N_i)$, and $e_i(w,a,N_i)$ denote the individual's optimal choice of $c_i$, $\ell_i$, and $e_i$, respectively.[6] Our first result shows that $\ell_i(w,a,N_i)$ is decreasing in $w$ while $e_i(w,a,N_i)$ is non-increasing.

**Lemma 1.** *The optimal leisure level, $\ell_i(w,a,N_i)$, is independent of $H^s$ and decreasing in $w$. The optimal effort level, $e_i(w,a,N_i)$, is independent of $H^s$, non-increasing in $w$ everywhere, and decreasing in $w$ when the effort constraint is not binding.*

*Proof.* See Appendix. □

Intuitively, the result follows because an increase in wage makes the opportunity cost of not working greater, putting downward pressure on the demand for leisure and the effort exerted in obtaining healthcare.[7] That rational individuals are willing to give up some consumption of healthcare to satisfy their other wants should not be surprising. Yet, we further emphasize that our model does not claim that every increase in wage will be accompanied by a corresponding decrease in HHR consumption. Our assumptions merely imply that every individual has a threshold wage level, which may be arbitrarily large or small, where she starts to make that trade-off. At any given wage, there may be individuals who exert full effort and others who do not, depending on whether the current wage is below or above their threshold level. Those who do not are already trading off some consumption of healthcare for other goods, and it is these individuals who will respond to increases in wage by further reducing their HHR consumption. Those who are currently exerting full effort may not respond to marginal changes in wages.

**Example 2.** For the utility function given in Example 1, the optimal effort level is

[5]We make two implicit assumptions in this formulation. First, the non-negativity constraints on consumption are not binding. Second, the individuals are myopic in that they take the accessibility level as given rather than recognize that their choices affect accessibility and behave accordingly. First is justified if the labor endowment is large, and second if the population is large.

[6]To ease the notation, the possible dependence of the solutions on the remaining parameters, such as $H^s$, is suppressed.

[7]Given the quasi-linearity of the utility function, any wealth effect arising from an increase in wage is absorbed by the demand for the composite good. Therefore, only the substitution effects influence the demands for leisure and healthcare.

given by:[8]

$$e_i(w,a,N_i) = \begin{cases} 1 & \text{if } \dfrac{\alpha_i^{\alpha_i}\beta_i^{1-\alpha_i}(aN_i)^{\beta_i}}{\bar{e}_i^{1-\alpha_i}} \geq w \\ \left(\dfrac{\alpha_i^{\alpha_i}\beta_i^{1-\alpha_i}(aN_i)^{\beta_i}}{\bar{e}_i^{\beta_i} w}\right)^{\frac{1}{1-\alpha_i-\beta_i}} & \text{otherwise.} \end{cases}$$

The graph of the optimal effort level as a function of wage is given in Figure 1.

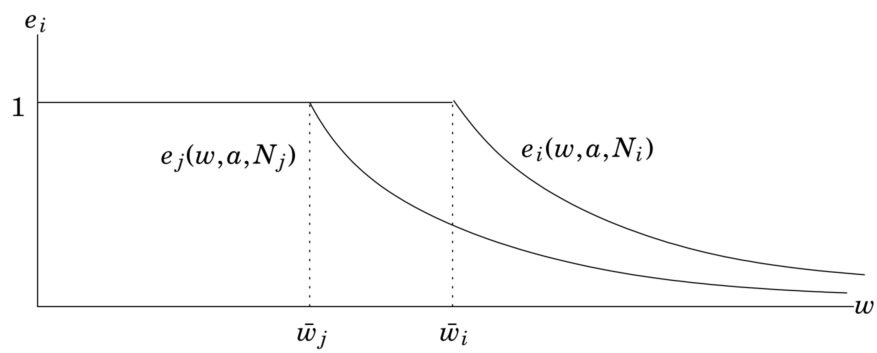


Figure 1: Optimal effort level ($\bar{e}_i = \bar{e}_j = \bar{e}$ and $N_i > N_j$)

At an interior solution, we have $\frac{\partial e_i(w,a,N_i)}{\partial N_i} > 0$ so that optimal effort is greater if the medical need is greater. The threshold wage is also increasing in the medical need. Thus, if we compare two individuals $i$ and $j$ who are identical except in their medical needs ($N_i > N_j$), then $i$'s threshold wage will be greater than $j$'s, and $i$ will also exert weakly greater effort than $j$ at all wage levels. □

We say that wage $w^*$ and allocation $\left((c_i^*, \ell_i^*, e_i^*) : i = 1,\ldots,I\right)$ is a *constrained equilibrium* of the economy with HHR supply $H^s$ if for all individual $i$,

$$\begin{aligned} c_i^* &= c_i(w^*, a(e^*, H^s), N_i), \\ \ell_i^* &= \ell_i(w^*, a(e^*, H^s), N_i), \\ \text{and} \quad e_i^* &= e_i(w^*, a(e^*, H^s), N_i); \end{aligned}$$

and

$$\sum_i \left(\bar{L}_i - \ell_i^* - e_i^*\right) = L^d(w^*), \tag{2}$$

$$\sum_i c_i^* = f(L^d(w^*)) - H^s, \tag{3}$$

$$\text{and} \quad \sum_i a(e^*, H^s) g_i(e_i^*, N_i) = H^s. \tag{4}$$

Conditions (2), (3), and (4) require that the labor, the composite good, and the HHR markets clear, respectively.

[8]The derivation is given in Appendix.

## 2.2 Planner's Problem

The planner wishes to supply HHR at a level that will maximize the aggregate welfare, which we take as the sum of the individual utilities. Aside from the supply level of HHR, the economy is a decentralized one in which individuals make utility maximizing choices. Therefore, we are ultimately interested in the planner's problem that considers only allocations that are achievable in a decentralized manner. That is, we wish to investigate the following problem:

$$\max_{H^s} \left\{ \sum_i u_i(c_i, \ell_i, e_i; a(e, H^s), N_i) : ((c_i, \ell_i, e_i) : i = 1, \ldots, I) \text{ is a} \right.$$
$$\left. \text{constrained equilibrium allocation under } H^s \right\}.$$

However, we begin by studying the more general problem of finding a centralized solution. We will later show that the solution to the general problem is the same as the decentralized solution.

To formulate the central planner's problem, we first define a social welfare function, $W$, that maps HHR supply level to a measure of social welfare. Say that allocation $((c_i, \ell_i, e_i) : i = 1 \ldots, I) \geq 0$ is *feasible* under $H^s$ if $\bar{L}_i - \ell_i - e_i \geq 0$ for all $i$ and

$$\sum_i c_i \;\leq\; f\left(\sum_i \left(\bar{L}_i - \ell_i - e_i\right)\right) - H^s.$$

In particular, a constrained equilibrium allocation is always feasible. We define $W(H^s)$ as the highest aggregate welfare that is attainable from allocations that are feasible under $H^s$:

$$W(H^s) \;=\; \max \left\{ \sum_i u_i(c_i, \ell_i, e_i; a(e, H^s), N_i) : ((c_i, \ell_i, e_i) : i = 1, \ldots, I) \text{ is} \right.$$
$$\left. \text{feasible under } H^s \right\}.$$

The planner's problem can now be written as:

$$\max_{H^s} \; W(H^s).$$

The following result shows that any feasible allocation that attains $W(H^s)$ must induce full access.

**Lemma 2.** *Suppose $((c_i, \ell_i, e_i) : i = 1, \ldots, I)$ is an allocation that is feasible under $H^s$ and attains $W(H^s)$ so that*

$$\sum_i u_i(c_i, \ell_i, e_i; a(e, H^s), N_i) \;=\; W(H^s).$$

*Then $a(e, H^s) = 1$.*

*Proof.* See Appendix. □

Using Lemma 2, we can reformulate the social welfare function as

$$W(H^s) = \max \left\{ \sum_i u_i(c_i, \ell_i, e_i; 1, N_i) : ((c_i, \ell_i, e_i) : i = 1, \ldots, I) \text{ is} \right.$$
$$\left. \text{feasible under } H^s \text{ and } \sum_i g_i(e_i, N_i) = H^s \right\}.$$

Our main result shows that a unique optimal supply level $H^*$ that maximizes social welfare exists and that it is achievable through decentralized means. Moreover, except in a very special case, $H^*$ is less than the aggregate need, $H^N = \sum_i N_i$. To see this, suppose the planner commits to providing 100% of the healthcare service that is demanded. Even with this guaranteed full access, however, an individual may not utilize the full amount of the healthcare prescribed by her medical need because each unit of healthcare she consumes entails trade-off with her other wants. Since provision of HHR is not without cost, the aggregate welfare will be maximized at the level of HHR that is exactly equal to the actual utilization. This is formally stated and shown in Theorem 3 and Corollary 4.

**Theorem 3.** *There is a unique HHR supply level $H^*$ that maximizes the social welfare function $W(H^s)$. Moreover, $W(H^*)$ is attained by the constrained equilibrium under $H^*$.*

*Proof.* Consider an economy unconstrained by any HHR supply restriction or healthcare cost burden. In such economy, $a \equiv 1$ and the individual's optimization problem is

$$\max_{c_i, \ell_i, e_i} \; c_i + \phi_i(\ell_i, e_i; 1, N_i) \quad \text{s.t.} \quad c_i \le \delta_i \pi_i(w) + w(\bar{L}_i - \ell_i - e_i)$$
$$\text{and} \quad e_i \le \bar{e}_i.$$

Since removal of the constant $-\theta_i H^s$ from the original budget constraint (1) does not affect the optimal leisure and effort choices, they are still given by the *constrained economy's* demand functions, $\ell_i(w, 1, N_i)$ and $e_i(w, 1, N_i)$. Individuals optimizing in this way, together with the firm, form a standard general equilibrium economy. Say that wage $w^*$ and allocation

$$\boldsymbol{x'} = \left( (c'_i, \ell_i(w^*, 1, N_i), e_i(w^*, 1, N_i)) : i = 1, \ldots, I \right)$$

is an *unconstrained equilibrium* if

$$\begin{aligned} \delta_i \pi_i(w^*) + w^* \left( \bar{L}_i - \ell_i(w^*, 1, N_i) - e_i(w^*, 1, N_i) \right) &= c'_i \quad \text{for all } i, \\ \sum_i \left( \bar{L}_i - \ell_i(w^*, 1, N_i) - e_i(w^*, 1, N_i) \right) &= L^d(w^*), \\ \text{and} \quad \sum_i c'_i &= f\left( L^d(w^*) \right). \end{aligned}$$

Because the preferences are strictly convex and the production function is strictly concave, a unique unconstrained equilibrium exists, and the equilibrium allocation $\boldsymbol{x}'$ is Pareto optimal by the first fundamental theorem of welfare.[9] Moreover, there is no other Pareto optimal allocation that involves leisure level different from $\ell_i(w^*,1,N_i)$ or effort level different from $e_i(w^*,1,N_i)$. To see this, note that the second fundamental theorem of welfare implies that, if allocation $\hat{\boldsymbol{x}} = ((\hat{c}_i,\hat{\ell}_i,\hat{e}_i) : i = 1,...,I)$ is Pareto optimal, then there must be wage $\hat{w}$ and transfers $T_1,T_2,...,T_I$, with $\sum T_i = 0$, such that $\hat{\boldsymbol{x}}$ is an unconstrained equilibrium allocation with budget constraint

$$c_i \;\leq\; T_i + \delta_i \pi(\hat{w}) + \hat{w}(\bar{L}_i - \ell_i - e_i).$$

Introducing constant $T_i$ in the budget constraint (1) does not affect the demand functions for leisure and effort, so $\hat{\ell}_i = \ell_i(\hat{w},1,N_i)$ and $\hat{e}_i = e_i(\hat{w},1,N_i)$. Suppose, towards contradiction, $\hat{\ell}_i \neq \ell_i(w^*,1,N_i)$ or $\hat{e}_i \neq e_i(w^*,1,N_i)$ so that $\hat{w} \neq w^*$. As seen in Lemma 1, $\ell_i(w,a,N_i)$ is decreasing in $w$ and $e_i(w,a,N_i)$ is non-increasing in $w$. Meanwhile, $L^d(w)$ is decreasing in $w$. Therefore, if $\hat{w} > w^*$ then

$$\begin{aligned}\sum_i \left(\bar{L}_i - \hat{\ell}_i - \hat{e}_i\right) \;&>\; \sum_i \left(\bar{L}_i - \ell_i(w^*,1,N_i) - e_i(w^*,1,N_i)\right)\\ &=\; L^d(w^*) \;>\; L^d(\hat{w}),\end{aligned}$$

and if $\hat{w} < w^*$ then

$$\begin{aligned}\sum_i \left(\bar{L}_i - \hat{\ell}_i - \hat{e}_i\right) \;&<\; \sum_i \left(\bar{L}_i - \ell_i(w^*,1,N_i) - e_i(w^*,1,N_i)\right)\\ &=\; L^d(w^*) \;<\; L^d(\hat{w}).\end{aligned}$$

Thus, the labor market cannot clear, which is a contradiction.

Let $\ell_i^* = \ell_i(w^*,1,N_i)$, $e_i^* = e_i(w^*,1,N_i)$, $H^* = \sum_i g_i(e_i^*,N_i)$, and $c_i^* = c_i' - \theta_i H^*$. Then allocation $\boldsymbol{x}^* = \left((c_i^*,\ell_i^*,e_i^*) : i = 1,...,I\right)$ is feasible under $H^*$, and, moreover, $w^*$ and $\boldsymbol{x}^*$ together form a constrained equilibrium of the economy with HHR supply level $H^*$.

Since every Pareto optimal allocation in the unconstrained economy must have individual $i$'s leisure and effort levels equal to $\ell_i^*$ and $e_i^*$, the same must hold for Pareto optimal allocations in the economy with HHR supply level $H^*$. This means that Pareto optimal allocations can differ only in the individual's consumption of the composite consumption good. Feasibility then implies that every Pareto optimal allocation in the economy with HHR level $H^*$ must have the same aggregate welfare as $\boldsymbol{x}^*$. Therefore,

$$\sum_i u_i(c_i^*,\ell_i^*,e_i^*;a(e^*,H^s),N_i) \;=\; W(H^*).$$

The above discussion also implies that every Pareto optimal allocation that is feasible under $H^s > H^*$ must yield lower aggregate welfare than $\boldsymbol{x}^*$ since it necessarily involves a a lower aggregate level of the composite good while having the same individual levels of leisure and effort. Furthermore, every allocation that is feasible under

[9] See, for example, Chapter 5 of Feldman and Serrano (2005) for a discussion of the welfare theorems in production economies.

$H^s < H^*$ must also yield lower aggregate utility than $\boldsymbol{x}^*$ since an allocation that is feasible under a HHR supply restriction is also feasible when no such restriction is imposed. Therefore, $H^*$ maximizes the social welfare function, $W(H^s)$. □

Theorem 3 implies that, to maximize aggregate welfare, the planner should provide HHR at the level equal to the actual utilization under full access. That is, $H^* = \sum_i g_i(e_i(w^*, 1, N_i), N_i)$. Therefore, supplying HHR at a level equal to the clinically determined needs, $H^N = \sum_i N_i$, cannot be optimal except in a very special situation where every individual's effort constraint is binding. We formally state this as Corollary 4.

**Corollary 4.** *Unless $e_i(w^*, 1, N_i) = \bar{e}_i$ for all $i$, we have $W(H^N) < W(H^*)$.*

*Proof.* If $e_i(w^*, 1, N_i) < \bar{e}_i$, then $g_i(e_i(w^*, 1, N_i), N_i) < N_i$ for some $i$. Therefore, $H^N \neq H^*$. □

**Remark.** As Fleurbaey and Schokkaert (2009) note, the literature on equity in healthcare delivery has "taken for granted" that justifiable inequalities, which are inequalities arising from differences in individual efforts, are "unproblematic," and may even be "desirable." Such view originated in normative economics under the doctrine of equality of opportunity. Corollary 4 can be seen as an expression of this idea within the purview of positive economics. This result shows that trying to "correct" justifiable inequalities by providing HHR beyond what individuals will utilize through their own effort will likely lead to a Pareto inefficient outcome. □

## 3 Conclusion

The needs-based models that are currently used for planning HHR in several countries use a definition of "needs" that includes only the medical circumstances of the individuals and not personal preferences or other socio-economic factors. In this paper, we examined whether planning based on such a narrow notion of needs will maximize social welfare. Specifically, we considered the social planner's problem of providing an optimal supply of HHR in a publicly funded healthcare system, and showed that over-supply of HHR is likely if the planner provides a supply equal to the aggregate need without taking utilization into consideration. Our result suggests that HHR planning should track the healthcare system as the access gradually improves since, even if healthcare is fully accessible, individuals may not fully utilize it to the degree prescribed by their medical circumstances. It is inefficient to produce HHR for needs that will not seek service.

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# A Appendix

**Lemma 1.** *The optimal leisure level, $\ell_i(w,a,N_i)$, is independent of $H^s$ and decreasing in $w$. The optimal effort level, $e_i(w,a,N_i)$, is independent of $H^s$, non-increasing in $w$ everywhere, and decreasing in $w$ when the effort constraint is not binding.*

*Proof.* The Lagrangian for an individual's optimization problem is

$$\mathscr{L} = c_i + \phi_i(\ell_i, e_i; a, N_i) + \lambda\left[\delta_i \pi(w) - \theta_i H^s + w\left(\bar{L}_i - \ell_i - e_i\right) - c_i\right] + \mu[\bar{e}_i - e_i].$$

Letting $c_i^*$, $\ell_i^*$, and $e_i^*$ denote the solutions and $h_i^* = a g_i(e_i^*, N_i)$, the first order conditions are

$$\begin{aligned} 1-\lambda &= 0 \\ \frac{\partial \phi_i(\ell_i^*, h_i^*)}{\partial \ell_i} - \lambda w &= 0 \\ \frac{\partial \phi_i(\ell_i^*, h_i^*)}{\partial h_i}\frac{\partial h_i(e_i^*)}{\partial e_i} - \lambda w - \mu &= 0 \\ \lambda\left[\delta_i \pi(w) - \theta_i H^s + w\bar{L}_i - w\ell_i^* - w e_i^* - c_i^*\right] &= 0 \\ \mu[\bar{e}_i - e_i^*] &= 0. \end{aligned}$$

We first look for the solution in which the effort constraint is not binding. Then $\mu = 0$, and the first order conditions reduce to

$$\frac{\partial \phi_i(\ell_i^*, h_i^*)}{\partial \ell_i} - w = 0 \tag{5}$$

$$\frac{\partial \phi_i(\ell_i^*, h_i^*)}{\partial h_i}\frac{\partial h_i(e_i^*)}{\partial e_i} - w = 0 \tag{6}$$

$$\delta_i \pi(w) - \theta_i H^s + w\bar{L}_i - w\ell_i^* - w e_i^* - c_i^* = 0.$$

Call the implicit functions defined by Equations (5) and (6), $F_1$ and $F_2$, respectively. The implicit function theorem yields:

$$\begin{bmatrix} \frac{\partial \ell_i^*}{\partial w} \\ \frac{\partial e_i^*}{\partial w} \end{bmatrix} = -\begin{bmatrix} \frac{\partial F_1}{\partial \ell_i} & \frac{\partial F_1}{\partial e_i} \\ \frac{\partial F_2}{\partial \ell_i} & \frac{\partial F_2}{\partial e_i} \end{bmatrix}^{-1} \begin{bmatrix} \frac{\partial F_1}{\partial w} \\ \frac{\partial F_2}{\partial w} \end{bmatrix} = -\begin{bmatrix} \frac{\partial^2 \phi_i}{\partial \ell_i^2} & \frac{\partial^2 \phi_i}{\partial \ell_i \partial h_i}\frac{\partial h_i}{\partial e_i} \\ \frac{\partial^2 \phi_i}{\partial h_i \partial \ell_i}\frac{\partial h_i}{\partial e_i} & \frac{\partial^2 \phi_i}{\partial h_i^2}\left(\frac{\partial h_i}{\partial e_i}\right)^2 + \frac{\partial \phi_i}{\partial h_i}\frac{\partial^2 h_i}{\partial e_i^2} \end{bmatrix}^{-1} \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$= \frac{1}{\frac{\partial^2 \phi_i}{\partial \ell_i^2}\left[\frac{\partial^2 \phi_i}{\partial h_i^2}\left(\frac{\partial h_i}{\partial e_i}\right)^2 + \frac{\partial \phi_i}{\partial h_i}\frac{\partial^2 h_i}{\partial e_i^2}\right] - \left(\frac{\partial^2 \phi_i}{\partial \ell_i \partial h_i}\frac{\partial h_i}{\partial e_i}\right)^2} \begin{bmatrix} \frac{\partial^2 \phi_i}{\partial h_i^2}\left(\frac{\partial h_i}{\partial e_i}\right)^2 + \frac{\partial \phi_i}{\partial h_i}\frac{\partial^2 h_i}{\partial e_i^2} & -\frac{\partial^2 \phi_i}{\partial h_i \partial \ell_i}\frac{\partial h_i}{\partial e_i} \\ -\frac{\partial^2 \phi_i}{\partial \ell_i \partial h_i}\frac{\partial h_i}{\partial e_i} & \frac{\partial^2 \phi_i}{\partial \ell_i^2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{\left[\frac{\partial^2 \phi_i}{\partial \ell_i^2}\frac{\partial^2 \phi_i}{\partial h_i^2} - \left(\frac{\partial^2 \phi_i}{\partial \ell_i \partial h_i}\right)^2\right]\left(\frac{\partial h_i}{\partial e_i}\right)^2 + \frac{\partial^2 \phi_i}{\partial \ell_i^2}\frac{\partial \phi_i}{\partial h_i}\frac{\partial^2 h_i}{\partial e_i^2}} \begin{bmatrix} \frac{\partial^2 \phi_i}{\partial h_i^2}\left(\frac{\partial h_i}{\partial e_i}\right)^2 + \frac{\partial \phi_i}{\partial h_i}\frac{\partial^2 h_i}{\partial e_i^2} - \frac{\partial^2 \phi_i}{\partial h_i \partial \ell_i}\frac{\partial h_i}{\partial e_i} \\ -\frac{\partial^2 \phi_i}{\partial \ell_i \partial h_i}\frac{\partial h_i}{\partial e_i} + \frac{\partial^2 \phi_i}{\partial \ell_i^2} \end{bmatrix}.$$

Since $\phi_i$ is negative definite in $(\ell_i, h_i)$, $\det(D^2_{(\ell_i,h_i)}\phi_i) > 0$. So we have

$$\begin{bmatrix} \frac{\partial \ell_i^*}{\partial w} \\ \\ \frac{\partial e_i^*}{\partial w} \end{bmatrix} = \frac{1}{\underbrace{\det\left(D^2_{(\ell_i,h_i)}\phi_i\right)}_{(+)}\underbrace{\left(\frac{\partial h_i}{\partial e_i}\right)^2}_{(+)} + \underbrace{\frac{\partial^2\phi_i}{\partial \ell_i^2}}_{(-)}\underbrace{\frac{\partial\phi_i}{\partial h_i}}_{(+)}\underbrace{\frac{\partial^2 h_i}{\partial e_i^2}}_{(\le 0)}} \begin{bmatrix} \frac{\partial^2\phi_i}{\partial h_i^2}\left(\frac{\partial h_i}{\partial e_i}\right)^2 + \frac{\partial\phi_i}{\partial h_i}\frac{\partial^2 h_i}{\partial e_i^2} - \frac{\partial^2\phi_i}{\partial h_i \partial \ell_i}\frac{\partial h_i}{\partial e_i} \\ \\ -\frac{\partial^2\phi_i}{\partial \ell_i \partial h_i}\frac{\partial h_i}{\partial e_i} + \frac{\partial^2\phi_i}{\partial \ell_i^2} \end{bmatrix}$$

$$= \frac{1}{(+)} \begin{bmatrix} \underbrace{\frac{\partial^2\phi_i}{\partial h_i^2}}_{(-)}\underbrace{\left(\frac{\partial h_i}{\partial e_i}\right)^2}_{(+)} + \underbrace{\frac{\partial\phi_i}{\partial h_i}}_{(+)}\underbrace{\frac{\partial^2 h_i}{\partial e_i^2}}_{(\le 0)} - \underbrace{\frac{\partial^2\phi_i}{\partial h_i \partial \ell_i}}_{(+)}\underbrace{\frac{\partial h_i}{\partial e_i}}_{(+)} \\ \\ -\underbrace{\frac{\partial^2\phi_i}{\partial \ell_i \partial h_i}}_{(+)}\underbrace{\frac{\partial h_i}{\partial e_i}}_{(+)} + \underbrace{\frac{\partial^2\phi_i}{\partial \ell_i^2}}_{(-)} \end{bmatrix} \ll 0.$$

Therefore, both $\ell_i^*$ and $e_i^*$ are strictly decreasing in $w$ when the effort constraint is not binding.

We now look for the solution in which the effort constraint is binding; that is, $e_i^* = \bar{e}_i$ and $h_i^* = a g_i(\bar{e}_i, N_i)$. Since $\lambda = 1$, the first order conditions now reduce to

$$\frac{\partial\phi_i(\ell_i^*, h_i^*)}{\partial \ell_i} - w = 0 \quad (7)$$

$$\frac{\partial\phi_i(\ell_i^*, h_i^*)}{\partial h_i}\frac{\partial h_i(\bar{e}_i)}{\partial e_i} - w - \mu = 0 \quad (8)$$

$$\delta_i \pi(w) - \theta_i H^s + w\bar{L}_i - w\ell_i^* - w\bar{e}_i - c_i^* = 0.$$

Implicitly differentiating Equation (7) yields

$$\frac{\partial \ell_i^*}{\partial w} = \frac{1}{\frac{\partial^2\phi_i(\ell_i^*, h_i^*)}{\partial \ell_i^2}} < 0.$$

So, $\ell_i^*$ is decreasing in $w$ and $e_i^* \equiv \bar{e}_i$ is non-increasing when the effort constraint is binding.

Lastly, we note that $H^s$ is present only in the equations that determine $c_i(w, a, N_i)$. Therefore, $\ell_i(w, a, N_i)$ and $e_i(w, a, N_i)$ are independent of $H^s$. □

**Example 2.** *Consider the utility function given in Example 1:*

$$u_i(c_i, \ell_i, e_i; a, N_i) = c_i + \ell_i^{\alpha_i}\left(\frac{aN_i e_i}{\bar{e}_i}\right)^{\beta_i},$$

*where $\alpha_i > 0$, $\beta_i > 0$, and $\alpha_i + \beta_i < 1$. The optimal effort level for this utility function is given by:*

$$e_i(w, a, N_i) = \begin{cases} \bar{e}_i & \text{if } \frac{\alpha_i^{\alpha_i}\beta_i^{1-\alpha_i}(aN_i)^{\beta_i}}{\bar{e}_i^{1-\alpha_i}} \ge w \\ \\ \left(\frac{\alpha_i^{\alpha_i}\beta_i^{1-\alpha_i}(aN_i)^{\beta_i}}{\bar{e}_i^{\beta_i} w}\right)^{\frac{1}{1-\alpha_i-\beta_i}} & \text{otherwise.} \end{cases}$$

*Proof.* The Lagrangian for the individual's optimization problem is

$$\mathscr{L} = c_i + \ell_i^{\alpha_i}\left(\frac{aN_ie_i}{\bar{e}_i}\right)^{\beta_i} + \lambda\left[\delta_i\pi_i(w) - \theta_iH^s + w\left(\bar{L}_i - \ell_i - e_i\right) - c_i\right] + \mu[\bar{e}_i - e_i].$$

The first order conditions are

$$\begin{aligned}
1-\lambda &= 0\\
\alpha_i\ell_i^{\alpha_i-1}\left(\frac{aN_ie_i}{\bar{e}_i}\right)^{\beta_i} - \lambda w &= 0\\
\beta_i\ell_i^{\alpha_i}\left(\frac{aN_i}{\bar{e}_i}\right)^{\beta_i} e_i^{\beta_i-1} - \lambda w - \mu &= 0\\
\lambda\left[\delta_i\pi_i(w) - \theta_iH^s + w\bar{L}_i - w\ell_i - we_i - c_i\right] &= 0\\
\mu[\bar{e}_i - e_i] &= 0.
\end{aligned}$$

We look for the solution in which the effort constraint is binding. That is, the case where $e_i(w,a,N_i) = \bar{e}_i$. Since $\lambda = 1$, the first order conditions reduce to

$$\begin{aligned}
\alpha_i\ell_i^{\alpha_i-1}(aN_i)^{\beta_i} &= w &\quad (9)\\
\beta_i\ell_i^{\alpha_i}(aN_i)^{\beta_i}\bar{e}_i^{-1} &= w+\mu &\quad (10)\\
\delta_i\pi_i(w) - \theta_iH^s + w\bar{L}_i - w\ell_i - we_i - c_i &= 0.
\end{aligned}$$

Simplifying Equation (9) yields

$$\ell_i^{1-\alpha_i} = \left(\frac{\alpha_i(aN_i)^{\beta_i}}{w}\right) \quad\Rightarrow\quad \ell_i = \left(\frac{\alpha_i(aN_i)^{\beta_i}}{w}\right)^{\frac{1}{1-\alpha_i}}.$$

Next, dividing Equation (10) by Equation (9) yields

$$\frac{\beta_i\ell_i}{\alpha_i\bar{e}_i} = \frac{w+\mu}{w} \quad\Rightarrow\quad \mu = \frac{w\beta_i\ell_i}{\alpha_i\bar{e}_i} - w.$$

Therefore $\mu \geq 0$ if and only if

$$\begin{aligned}
\left(\frac{w\beta_i}{\alpha_i\bar{e}_i}\right)\left(\frac{\alpha_i(aN_i)^{\beta_i}}{w}\right)^{\frac{1}{1-\alpha_i}} &\geq w\\
\frac{\beta_i^{1-\alpha_i}}{(\alpha_i\bar{e}_i)^{1-\alpha_i}}\left(\frac{\alpha_i(aN_i)^{\beta_i}}{w}\right) &\geq 1\\
\frac{\alpha_i^{\alpha_i}\beta_i^{1-\alpha_i}(aN_i)^{\beta_i}}{\bar{e}_i^{1-\alpha_i}} &\geq w.
\end{aligned}$$

We next look for the solution where effort constraint is not binding. Then $\mu = 0$, and the first order conditions now reduce to

$$\begin{aligned}
\alpha_i\ell_i^{\alpha_i-1}\left(\frac{aN_ie_i}{\bar{e}_i}\right)^{\beta_i} &= w &\quad (11)\\
\beta_i\ell_i^{\alpha_i}\left(\frac{aN_i}{\bar{e}_i}\right)^{\beta_i} e_i^{\beta_i-1} &= w &\quad (12)\\
\delta_i\pi_i(w) - \theta_iH^s + w\bar{L}_i - w\ell_i - we_i - c_i &= 0.
\end{aligned}$$

Dividing Equation (12) by Equation (11) yields

$$\frac{\beta_i \ell_i}{\alpha_i e_i} = 1 \quad \Rightarrow \quad \ell_i = \frac{\alpha_i}{\beta_i} e_i.$$

Substitute this into Equation (11) to obtain

$$\alpha_i \left(\frac{\alpha_i}{\beta_i} e_i\right)^{\alpha_i - 1} \left(\frac{a N_i e_i}{\bar{e}_i}\right)^{\beta_i} = w \quad \Rightarrow \quad \frac{\alpha_i^{\alpha_i}}{\beta_i^{\alpha_i - 1}} \left(\frac{a N_i}{\bar{e}_i}\right)^{\beta_i} e_i^{\alpha_i + \beta_i - 1} = w.$$

Therefore,

$$e_i(w, a_i, N_i) = \left(\frac{\alpha_i^{\alpha_i} \beta_i^{1-\alpha_i} (a N_i)^{\beta_i}}{\bar{e}_i^{\beta_i} w}\right)^{\frac{1}{1-\alpha_i-\beta_i}}. \qquad \square$$

**Lemma 2.** *Suppose* $((c_i, \ell_i, e_i) : i = 1, \ldots, I)$ *is an allocation that is feasible under* $H^s$ *and attains* $W(H^s)$ *so that*

$$\sum_i u_i(c_i, \ell_i, e_i; a(e, H^s), N_i) = W(H^s).$$

*Then* $a(e, H^s) = 1$.

*Proof.* Let $((c_i, \ell_i, e_i) : i = 1, \ldots, I)$ attain $W(H^s)$. Suppose, towards contradiction, that

$$a(e, H^s) = \min \left\{1, \frac{H^s}{\sum_i g_i(e_i, N_i)}\right\} = \frac{H^s}{\sum_i g_i(e_i, N_i)} < 1.$$

For each $i$, let $\tilde{\ell}_i = \ell_i$, and let $\tilde{e}_i$ be such that $g_i(\tilde{e}_i, N_i) = a(e, H^s) g_i(e_i, N_i)$. Then

$$a(\tilde{e}, H^s) = \min \left\{1, \frac{H^s}{\sum_i g_i(\tilde{e}_i, N_i)}\right\} = \min \left\{1, \frac{H^s}{\sum_i a(e, H^s) g_i(e_i, N_i)}\right\} = 1.$$

Since $g_i(\tilde{e}_i, N_i) < g_i(e_i, N_i)$ and $g_i$ is an increasing function of effort, $\tilde{e}_i < e_i$ for all $i$. Thus,

$$f\left(\sum_i (\bar{L}_i - \ell_i - \tilde{e}_i)\right) - H^s > f\left(\sum_i (\bar{L}_i - \ell_i - e_i)\right) - H^s = \sum_i c_i.$$

Now, for each individual $i$, let

$$\tilde{c}_i = c_i + \frac{f\left(\sum_i (\bar{L}_i - \ell_i - \tilde{e}_i)\right) - H^s - \sum_i c_i}{I}.$$

Then

$$\sum_i \tilde{c}_i = \sum_i c_i + \sum_i \left(\frac{f\left(\sum_i (\bar{L}_i - \ell_i - \tilde{e}_i)\right) - H^s - \sum_i c_i}{I}\right) = f\left(\sum_i (\bar{L}_i - \ell_i - \tilde{e}_i)\right) - H^s.$$

So allocation $((\tilde{c}_i, \tilde{\ell}_i, \tilde{e}_i) : i = 1, \ldots, I)$ is feasible under $H^s$. Moreover, for all $i$,

$$\tilde{c}_i + \phi_i(\tilde{\ell}_i, \tilde{e}_i; a(\tilde{e}, H^s), N_i) > c_i + \phi_i(\ell_i, e_i; a(e, H^s), N_i),$$

since $\tilde{c}_i > c_i$. However, this contradicts the assumption that $((c_i, \ell_i, e_i) : i = 1, \ldots, I)$ attains $W(H^s)$. Therefore, $a(e, H^s) = 1$. $\square$