Child-raising cost and fertility from a contest perspective*

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Abstract

We model parents' fertility and child-raising spending decisions as a Tullock contest with budget constraints and prizes that depend on relative efforts. We show that if the consequences of failure and the intensity of competition are sufficiently high, some potential parents forego having children in the resulting equilibrium. Moreover, parents having children prefer to have only one child and allocate all of their resources to raising that child rather than have multiple children. The equilibrium is consistent with the recent East Asian fertility experience and, more broadly, shows that competitive pressure, when combined with a high degree of inequality or poor social protection, can lead to an overaccumulation of human capital and low fertility.

Key words: child-raising cost, education, fertility, quantity-quality trade-off, competition, Tullock contest

JEL Classification: D70, D91, I21, J13

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1 Introduction

Malthusian fears of runaway population growth notwithstanding, low fertility, rather than high fertility, has been the dominant concern in much of the developed world since the 1990s. The concern is most acute in the so-called "lowest-low fertility" countries that have total fertility rates (TFR) of 1.3 or below. At that TFR, assuming childbirth at age 30 and no migration, the population can be halved in a mere 44 years (Toulemon 2011). Given the specter of such a population implosion, it is not surprising that many countries have tried to boost their fertility rates by implementing various pronatalist policies, including broader access to childcare, better parental-leave benefits, and direct and indirect cash support programs. Although the complex nature of fertility decisions makes determining the effectiveness of those policies difficult in general, a number of studies have concluded that cash support programs at least positively affect fertility in Australia, Europe and North America (d'Addio and d'Ercole 2005; Drago et al. 2011; Ermisch 1988; Milligan 2005; Whittington 1992; Whittington et al. 1990; Zhang et al. 1994).[1] In contrast, a consensus has risen that cash supports have been a failure in East Asian countries that have adopted them (Jones and Hamid 2015; Lee and Choi 2015; McDonald 2006). Indeed, while many of the lowest-low fertility countries began to see a rise in TFR since year 2000, East Asia has been the glaring exception.[2] According to Goldstein, Sobotka and Jasilioniene (2009), the number of lowest-low fertility countries and territories fell from 21 in 2003 to five by 2008. Yet, strikingly, four out of the remaining five were East Asian: Hong Kong, Singapore, South Korea and Taiwan, with the only exception being Moldova. Moreover, Japan barely escaped the lowest-low label with a TFR of 1.4. While the numbers and the identities of the lowest-low fertility countries have changed slightly from year to year since 2008, it nevertheless remains true that fertility rates in East Asia have ranked close to the bottom throughout that period.

Consistent and near universal ultra-low fertility in East Asia suggests that cultural or institutional features unique to the region may be in play. Accordingly, many studies have pointed out that the burdens of childcare, education, housing, employment uncertainty and gender inequality all are greater in East Asia and contribute to its low fertility (Choe and Retherford 2009; Frejka et al. 2010; Gauthier 2016; Yoon 2016). One of the most critical constraints on fertility, though, appears to be an ordinary financial one, namely the high cost of raising a child, including education (Anderson and Kohler 2013; Choe and Retherford 2009; Gauthier 2016; Jones and Hamid 2015). Thus, a natural question that arises is why cash support programs have been ineffective in spurring fertility in East Asia, especially when they seem effective elsewhere. Although one simply may suppose that the amount of cash support has been too small to significantly defray the cost of raising a child, such an explanation seems at odds with the substantial support some East Asian nations provide

[1] One exception to this strand of literature is Kalwij (2010), who concludes that cash grants affected the timing of the births but not the total fertility in Western Europe.

[2] Although Singapore is located geographically in Southeast Asia, we consider it to be East Asian because of its culture. In addition, mainland China is excluded from the discussion since its legal restrictions on fertility make unclear the extent to which its low fertility is from parental choice or government-imposed.

to parents. For example, up to one-third of the cost of raising a child from birth to adulthood is subsidized by the Singaporean government (Jones and Hamid 2015), yet Singapore's TFR has not risen above 1.3 since 2003. Consequently, explaining East Asian fertility requires identifying the factors that limit fertility through parents' budget constraints, but at the same time make fertility unresponsive to their relaxation. This paper proposes the hypercompetitive environment in which the children are raised in East Asia as a possible key factor. While the ubiquity and the fierce intensity of competition in East Asia have been noted by a number of researchers as an important source of its low fertility (Anderson and Kohler 2013; Choe and Retherford 2009; Tan et al. 2016), competition has not been studied formally in the fertility and child-raising context. Therefore, we develop a theoretical model to demonstrate rigorously that competitive pressures, when combined with high income inequality or poor social protection, can drive up child-raising costs, constrain fertility and render cash support ineffective.

At its core, competition turns fertility and child-raising spending decisions from an individual decision-making problem into a more complex social one that is interdependent on each other's choices. To see that point, suppose that parents are altruistic and educate their child to produce high-quality progeny because a high-quality child is more likely to have a "successful life". If no competitive element is in play, the likelihood of success depends only on the child's own quality and not on the quality of other children. In contrast, suppose that the number of children who can succeed is constrained, and success is determined by a contest in which a higher-quality child has a better chance of succeeding. Then it no longer is the absolute quality of the child, but her quality relative to the others that is crucial because no matter how high the quality of a child is in absolute terms, the child is not likely to succeed if other children with even higher qualities also compete. That consideration raises the possibility that parents always will strive to outspend each other in a bid to increase the likelihood of their child's success until they exhaust their budgets. That is, competition may distort Becker's quantity-quality tradeoff for children (Becker 1960; Becker and Lewis 1973) into a human capital arms race that forces parents constantly to forgo quantity in favor of quality. To study that issue, we start with Tullock's (1980) model of competition wherein the probability of a child's success roughly is proportional to how much the child's parents spend relative to the other parents. Since the value of success, or failure, likewise should depend on the child's quality relative to the average, we then depart from the standard Tullockian framework and allow the parents' utility to depend on their spending relative to others.[3]

We show that in such an environment whether competitive pressure leads to budget-exhausting equilibrium hinges on how severely children's failure affects parents' utility. When the consequences of failure are modest because, for example, the level of social protection is generous or income inequality is low, an interior, non-budget-exhausting equilibrium exists. To the extent that fertility is limited by a lack

[3]While investigating the effects of risk-aversion on the contest participants' effort levels, Skaperdas and Gan (1995) show that budget exhaustion can occur in equilibrium if certain conditions on the class of success probabilities and utility functions they consider are satisfied. In this paper, we have taken a different approach and allow the utilities to depend on relative effort levels, as we find it more compelling in the current context.

of resources, that is the equilibrium when programs that relax parents' budget constraints, such as cash support and education subsidies, can increase fertility. If the consequences of failure are severe, though, parents exhaust their child-raising budgets in equilibrium, no matter how large their budgets are. If in addition the competitive pressure is sufficiently strong, some potential parents will forego having children in the equilibrium, and those who do have children will have only one and allocate all of their resources to raising that child rather than divide it among multiple children and risk failures. Relaxing the budget constraint will not have a positive effect on fertility in such an equilibrium because any additional resources will go into making the existing child more competitive, rather than being used to produce more children. Since that equilibrium occurs when both the level of competition is vigorous and the impact of a child's failure on parents' utility is large, it may explain why cash support that seems effective elsewhere appears to be ineffective in East Asian countries, which have limited social safety nets compared to other developed nations.

Our contest framework assumes relative outcomes, or rankings, as the motivating force for inducing effort. That assumption is supported by the relative deprivation literature, which has accumulated substantial evidence that "interpersonal comparisons of income" have a significant effect on perceptions of well-being and observable behavior (Sorger and Stark 2013).[4] Although research on cross-country comparisons of rank sensitivity is sparse, the few studies that do exist find that East Asia is more rank-conscious in general than other developed regions. Chung and Mallery (1999), for example, conclude that East Asia's "collectivistic" culture generates a stronger desire to make interpersonal comparisons relative to the more "individualistic" West. Moreover, they find that East Asians tend to make upward social comparisons, which make them less happy and presumably induce them to exert greater effort to catch up. Similar findings have been reported by Clark, Senik and Yamada (2013), Lee and Ohtake (2018), and White and Lehman (2005). As we detail in Remark 2.1 below, such general sensitivity to rank becomes extreme when it comes to evaluating the university from which an individual has graduated. A substantial literature observes that "education credentialism", in which individuals are judged by the rankings of their alma maters, is woven deeply into East Asian society. It not only is critical for financial success, but also directly affects one's social standing in both formal and informal settings (Iga 1981; Lee and Brinton 1996; Sorensen 1994; Tan et al. 2016).[5] Sorensen (1994), for example, states that the rankings of the university individuals attend determine their "social prestige for the rest of their life" and that rare individuals who become successful financially without having the proper educational pedigrees cannot gain high social status. To emphasize further, Sorensen notes that in Korea a person with a "better" education will earn more for doing exactly the same work as others and that, what is more important, that wage advantage is perceived as just.

Our work relates to that of Doepke and Zilibotti (2017), who show that parental decisions, in particular the choice of parenting style, are driven by economic factors

[4] Clark (2017) provides a survey of the relevant empirical evidence.

[5] Those observations are tempered somewhat in mainland China owing to its political system and the lingering effects of cultural revolution.

like inequality. By introducing a competitive element into the quantity-quality tradeoff framework, we show that inequality and inadequate social protections can affect human capital accumulation and fertility as well. Our results also bring a new perspective to the debate about the existence of the quantity-quality tradeoff itself. Although the theory of the tradeoff is accepted widely, the evidence for it has been mixed. Empirical work using data from Israel (Angrist et al. 2010), Norway (Black et al. 2005) and the United States (Cáceres-Delpiano 2006) have not found a negative relationship between family size and child quality. In contrast, other works that use data from China (Li et al. 2008; Rosenzweig and Zhang 2009), India (Rosenzweig and Wolpin 1980) and South Korea (Lee 2008) have found a negative relationship. Those results appear largely consistent with our model's prediction that the tradeoff will be more pronounced in societies with strong competition and weak social safety nets.

At the theoretical level, our paper contributes to the contest literature by analyzing the effects of budget constraints in a novel setting. In general, incorporating budget constraints is important not only for the realism it provides, but also for the difference in the resulting equilibrium behavior. For example, Che and Gale (1997) have shown that contestants may exert more effort in the Tullock contest if a budget constraint is imposed; Che and Gale (1998) and Gavious, Moldovanu and Sela (2002) obtained similar results for all-pay auctions. A number of authors have extended the analysis of budget constraints to multi-stage Tullock contests, in which contestants decide how to allocate their resources across multiple contests. In particular, Megidish and Sela (2014) showed that when budget constraints bind, contestants spend the same amounts of resources at each stage of a sequential two-stage contest. An earlier work by Klumpp and Polborn (2006) used numerical methods to demonstrate that contestants exert greater efforts in the early stages of a sequential contest with a more general contest success function. Like those models, parents herein may participate in multiple contests, one for each child. However, since parents typically make their subsequent parity decisions before the contest involving the first child is resolved, we treat the stages as simultaneous. In that regard, our setting is simpler than contemplated in previous sequential contests, which assume that the contestants observe the outcomes of earlier stages before choosing their later-stage efforts. However, our setting is more complex in that the contest's prizes, as well as the number of contests in which each parent engages, are endogenous. Thus, our model differs from the simultaneous version of the multi-stage game that also was studied by Klumpp and Polborn (2006).

The remainder of the paper is organized as follows. Section 2 presents our model of competition and, as a preliminary study, analyzes its effect on child-raising costs only. We identify the parameters that represent the degree of competition and the severity of failure, respectively, and show that an interior equilibrium, which does not exhaust budgets, exists when the parameters are below certain threshold levels. We then show that when the severity of failure parameter is above the previously found threshold, everyone exhausting their budgets is the unique symmetric equilibrium. Section 3 then considers the effect of competition on fertility by giving parents a choice between having zero, one, or two children. Our main result shows that if both parameters exceed certain thresholds, then the unique symmetric equilibrium involves some couples not having any children, while the rest have only one child and exhaust their resources on their sole child. The threshold for the severity of failure parameter is the same as in Section 2, while the threshold for competition is slightly higher. Section 4 concludes. All proofs and supporting propositions are given in the Appendix.

2 Competition and child-raising cost

We put aside the issue of fertility in this section and consider only the implications of competition on child-raising cost. Consequently, we assume that all parents have exactly one child, and the only remaining decision is how much to spend on raising their child, which determines the child's quality. Therefore, the term "child-raising cost" should be interpreted broadly as an amalgamation of pecuniary and non-pecuniary efforts exerted in raising a child from birth to adulthood, including education. We assume that children grow up in a competitive world, where the likelihood of success depends on their quality relative to other children. Thus, whether a child succeeds or not ultimately is determined by her parents' spending relative to the other parents, although the existence of random elements means that determination is not perfect.

More specifically, we consider a group of N identical parenting couples who choose simultaneously how much to spend on raising their children. Let c_i denote couple i's spending, and let $\boldsymbol{c} = (c_1, ..., c_N)$ be the profile of spending for the group. We assume that $c_i \in [c_{\min}, c_{\max}]$, where $c_{\max} > c_{\min} > 0$. Thus, everyone faces the same budget constraint, and the minimum spending level is positive. The competition among the children is modeled abstractly as a Tullock contest, with either success or failure as the two possible outcomes. In particular, we assume that spending c_i produces a child of quality c_i, and the probability that the child will succeed is given by the following.[6]

$$P_i(\boldsymbol{c}) = \frac{c_i^r}{c_1^r + c_2^r + \cdots + c_N^r}, \quad \text{where } r > 0.$$

It is easy to see that the success probability is increasing in one's own spending and decreasing in other couple's spending. Two parameters reflect the intensity of competition in this model. The size of the group, N, clearly affects the level of competition.[7] In addition, parameter r determines how influential a child's quality and, hence, her parents' spending, is to her success. Therefore, larger r intensifies competition by creating a stronger incentive for parents to outspend their peers. For example, if $r = 0$, then $P_i(\boldsymbol{c}) = \frac{1}{N}$ for all $\boldsymbol{c}$. Since the probability of success is independent of the quality, parents have no incentive to spend more than the bare minimum. As r increases, parents' spending becomes more important. When $r = 1$, the competition is similar to a pure lottery, in which the probability of success is exactly proportional

[6] As shown in Fu and Lu (2012), that probability arises out of a noisy-ranking contest model in which contestants are ranked according to their effort multiplied by a noise term that follows an extreme value distribution. It also is equivalent to the "best-shot ranking rule", in which contestants are ranked according to their best performance in a number of independent trials.

[7] The expected number of successful children is normalized to be one in our model. Thus, rather than think of the parents in the model as representing the entire population, it may be more appropriate to view them as a peer group of parents whose children compete for one slot.

to one's spending relative to the aggregate spending. In the limit, $r = \infty$, and whoever has the highest quality, by however small a margin, wins the competition with probability one.

Remark 2.1. Parameter r is a structural parameter that is difficult to measure and compare across countries. However, we have strong reasons to believe that it is higher in East Asia than in developed Western nations (which we call the "West" here). For parents to believe that their investments will be effective, the link between investment and success must be clear. That link is very tight in East Asia because for most people both financial and social success require a degree from a highly ranked university, entrance to which is determined almost exclusively by highly competitive entrance examinations. It is difficult to overstate the importance of academic pedigrees for achieving success in East Asia. Describing Korea, but applying equally well to the rest of the region, the European Commission (2010) states that the ranking of an individual's alma mater, rather than the individual's experience or quality, is "perhaps the single most important factor in determining his or her life chances", and a large literature exists that attests to that connection.[8] Of course, knowing that academic pedigree determines success would not mean a high r if parents did not believe it was attainable. If only those with superb innate intelligences or the "right" family backgrounds can attend an elite university, the r-value will be low and the parents of less than brilliant children and those without good connections would simply give up. However, because East Asian entrance exams test acquired knowledge and skills, a nearly universal belief exists that effort largely determines success (Anderson Kohler 2013; Iga 1981; Marginson 2011; Sorensen 1994; Tan et al. 2016).

In contrast, the definition of success and the path to it typically are more varied and less formulaic in the West. Even in the United States, where the return to education generally is considered to be high, a 2016 survey has found that only 42% of the public believe that a university education is necessary for success and 57% believe that many ways are open to succeed without attending a university, let alone a high-ranking one (Public Agenda 2016).[9] It seems unlikely that parents holding those beliefs would think that additional tutoring or cramming ("hot-house") schools will increase their children's probabilities of success significantly. Even for parents who believe that a quality university education is important for success, how to obtain it is less clear than in East Asia. Since elite universities in the West often state that they evaluate candidates by their "whole packages", seeking individuals who are "promising", "well-rounded" or "passionate", it is hard for parents to know on what and how much to spend to make their children more promising, more well-rounded and more

[8]For example, Anderson and Kohler (2013), Iga (1981), Marginson (2011), Sorensen (1994) and Tan, Morgan, and Zagheni (2016) all observe that a degree from a prestigious university is necessary, and often sufficient, for success in East Asia. Existing labor market studies also confirm those observations. Van der Velden, van de Loo and Meng (2007) show that Japanese workers' earnings are correlated strongly with the rankings of their alma maters, but not with family backgrounds or acquired competencies, while the opposite holds for the Netherlands. Lee and Brinton (1996) find that the university ranking is important for obtaining prestigious jobs in Korea, but having higher individual abilities than other students from the same university are not.

[9]Attitudes in Canada are even more striking. A 2018 Ontario survey showed that only 15% of the public thought that a four-year university degree is necessary for success (Ontario Institute for Studies in Education 2018).

passionate than their potential competitors. Certainly, academic achievement also matters for admission in the West. However, the limited use of standardized grading systems in primary and secondary schools means that most times parents do not really know where their children stand academically at the national level and, thus, do not have a clear idea on the effects their investments eventually will have on their children's chances at university admissions. East Asia, on the other hand, measures and ranks students' scholastic performances relentlessly from an early age using a common national standard. Thus, students and their parents have very good assessments of where they stand at any given moment. Such information creates a constant competitive drive in which lower-ranked students struggle to catch up by employing more private tutors or attending better supplemental learning centers, while higher-ranked students strive to maintain their leads using the same means.

Differences in beliefs about r are reflected in how much East Asian and Western parents actually invest in their childrens' educations. Surveys have found that the percentage of students receiving private tutoring in primary and middle schools were, respectively, 87.9% and 72.5% in Korea (Kim 2010) and 73.8% and 65.6% in urban China (Xue and Ding 2009, as quoted in Zhang 2013). Surveys conducted in other East Asian economies yield similar results: 71.8% of 12th grade students in Hong Kong (Bray 2013), 65.2% of 9th grade students in Japan (Japanese Ministry of Education, as quoted in Bray 2013), and 72.9% of 7th grade students in Taiwan (Liu 2012) received private tutoring. Although studies of private tutoring in the West are few, existing surveys suggest that the scale is much more modest. Only 20% of Austrian parents had paid for tutoring in 2010, while one-third of Canadian parents had hired a private tutor at some point (Bray and Kobakhidze 2014). Peters, Carpenter, and Coleman (as cited in Bray and Kobakhidze 2014) found that 12% of primary students and 8% of secondary students in England were receiving private tutoring. In financial terms, children's pre-tertiary education takes up around 14.3% of all household spending in urban China (CIEFR-HS 2017), 6% in Japan, and 7% in Korea (Tan et al. 2016), which are significantly higher than the 1% spent in the European Union and 2.4% in the United States (European Commission 2010). □

Letting v_s be the utility from having a child who succeeds and v_f be the utility from one who fails, couple i's overall expected utility is

$$\pi_i(\boldsymbol{c}) = P_i(\boldsymbol{c})v_s + (1 - P_i(\boldsymbol{c}))v_f - c_i.$$

In the standard Tullock competition, v_f is zero and v_s is a positive constant that does not depend on $\boldsymbol{c}$.[10] However, in the current context in which competition is based on the quality of the children, the specification appears to be unreasonable since the consequences of failure to a child should be more severe if her quality is much lower than the average. Similarly, success should bring a larger return if her quality is much

[10] A number of scholars have extended the standard Tullock framework to incorporate non-constant success and failure utilities. For example, v_s depends positively on total effort in Chung (1996) and either positively or negatively in Damianov, Sanders and Yildizparlak (2018). The two-competitor model of Chowdhury and Sheremeta (2011) allows success and failure utilities to depend linearly on the competitors' efforts. In all-pay auctions, Kaplan, Luski, Sela, and Wettstein (2002) consider prizes that depend on the bidder's own bid as well as her type, while Sela (2017) considers two-stage auctions in which the prize in the second stage depends on the bidder's own bid in the first auction.

higher than the average. Thus, to the extent that parents' utilities are increasing in their child's welfare, it seems more appropriate to assume that the utility difference between success and failure, $v_s - v_f$, depends on the gap between the child's own quality and the average quality. To capture that in a tractable way, we could set v_f as a constant and have v_s be an increasing function of $c_i - \hat{c}_{-i}$, where $\hat{c}_{-i} = \sum_{j \neq i} c_j / (N-1)$ is the average quality of the other children, or set v_s as a constant and have v_f be a declining function of $\hat{c}_{-i} - c_i$. We have chosen the latter option in this paper since, as we discuss in Remark 2.2 below, it also fits naturally with the behavioral interpretation based on parental guilt. The following lists the assumptions that are placed on v_f, which we call the failure utility.

Assumption 1. Let $v_f(c, \hat{c})$ be the utility to a couple whose child fails when they had spent c, while the average spending, excluding the couple's, was $\hat{c}$. Then $v_f(c, \hat{c})$ is assumed to satisfy the following.

1. For all $\hat{c}$, $v_f(c, \hat{c})$ is twice differentiable, increasing and concave in c, and $v_f(c, \hat{c}) = 0$ at $c = \hat{c}$.
2. For all $\hat{c}$, $\left.\frac{\partial v_f(c,\hat{c})}{\partial c}\right|_{c=\hat{c}} = \lambda > 0$. □

Condition 1 normalizes the failure utility to be zero whenever a child's quality is the same as the average. It is negative when the quality is below the average and positive when it is above. The positiveness means that the consequences of failing in the competition are mitigated if the child's quality is high relative to the average. Condition 2 states that the extent to which $v_s - v_f(c, \hat{c})$ widens when parents make a marginal reduction in spending relative to the average is given by the parameter λ. As seen in Proposition A.1, Condition 2 and the concavity of v_f imply that $v_f(c, \hat{c})$ lies on or below the line $\lambda(c - \hat{c})$ for all $c \leq \hat{c}$, meaning that the utility difference $v_s - v_f(c, \hat{c})$ grows faster than, or at the rate equal to, $\lambda(\hat{c} - c)$ as c falls below $\hat{c}$. Thus, λ determines how severely a child's failure affects her parents' utility when they had spent less than the average.

Remark 2.2. Our primary interpretation of failure utility is the parents' altruistic concern for their child's welfare after failing. Thus, λ would be higher in countries without adequate social safety nets to dampen the negative consequences of failure or in less egalitarian societies where the difference between success and failure is wider. In that view, λ can be interpreted as a reflection of inequality in outcomes. In comparison to the other developed nations, the social safety nets in East Asia are lower. Social protection spending in Japan and Korea, for example, were 9.1% and 2.4% of their respective GDPs in 2001, while the average spending for the remaining OECD countries was 13.8% (World Bank 2006). Although comparable social protection spending for Singapore is not available, according to Asian Development Bank (2013), the social protection expenditure there was a mere 44.3% of South Korea's in 2009.

Although a perception exists that formal social safety nets are weak in East Asia because they are compensated by strong family support systems, it does not reflect current reality, as exemplified by elderly poverty in Japan and Korea. Korea often is regarded as having the most traditional Confucian culture, which places great emphasis on family ties and filial duty, but has the highest rate of old-age poverty among the OECD members. A 2015 report found that 49.6% of the Korean population aged 65 and above lived in relative poverty, which was four times greater than the OECD average of 12.4% (OECD 2015).[11] Japan, a country that prioritizes social harmony, did not fare much better and came in at seventh highest, with a 19.4% poverty rate (OECD 2015). Because the OECD's definition of poverty counts only market incomes and government transfers, the important question is what the poverty rate would be if intra-family supports were included. Using the 1996 figures, Kwon (2001) estimates that including private transfers would have pulled 33% of Korea's poorest elderly households above the poverty line. Because private transfers have been declining steadily since the 1980s, Kwon's estimate most likely overstates the current poverty-relieving power of private transfers. Nevertheless, using his estimate suggests that approximately 33% ($0.67 \times 49.6\%$) of elderly Koreans live in relative poverty even when private transfers are considered.

Because neither Korea nor Japan tracks absolute poverty for the elderly, the welfare of the elderly poor in absolute terms is difficult to obtain. However, it is likely to be bleak. The phenomenon of Japanese elderly who commit petty crimes just so that they can be cared for in prison has been publicized widely in the news media, as well as studied academically.[12] A particularly troubling reflection of elderly Korean's welfare is their suicide rate. Although Korea's suicide rate among the young is similar to the OECD average, the rate for the elderly is the highest in the OECD. For those 65 years old and above, the suicide rate is 72 per 100,000, which is more than three times the OECD average of 22 (OECD 2019). The OECD identifies poverty as a significant factor in elderly suicides in Korea; Statistics Korea (as quoted in Jones and Urasawa 2014) also found that "economic hardship" is the second most cited reason, after "disease and disability", by those who have considered suicide. Korea is an extreme example, but elderly suicide rates in other East Asian economies also are relatively high and reflect their grim reality. The figures for mainland China, Hong Kong, and Japan are 51, 33 and 29 per 100,000, respectively (OECD 2019); the comparable rate for Taiwan is 37 (Chan et al. 2011). To be clear, those statistics do not mean that family ties and support no longer are important in East Asia. As we have noted, parents in the region are willing to go to great lengths to support their children. Children also appear to give back what they can in return, as intra-family transfers still make up a significant fraction of financial resources for the elderly; however, they are insufficient for providing a basic standard of living for many. (Jones and Urasawa 2014; Kwon 2001).

Finally, in addition to our primary interpretation, we may suppose that failure (dis)utility also represents "regret", in which parents suffer guilt if they spend less than their peers and their child fails. That interpretation fits especially well with the parenting norm in East Asia, where parents see it as their "duty to provide their children with proper educational resources and support in order to produce successful

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[11] The OECD defines poverty relatively, as earning, inclusive of government transfers, less than or equal to 50% of the median income.

[12] See, for example, Nohara and Sharp (2013) and Sugie (2017).

and competitive children" (Anderson and Kohler 2013). Quoting Lee (2011), Anderson and Kohler note further that anyone not doing so is seen as "irresponsible" and "neglectful". Therefore, by both interpretations, we view East Asia as having a relatively high value of λ. □

Example 2.3. As Proposition A.2 in the Appendix shows, one example of failure utility that satisfies Assumption 1 is

$$v_f(c,\hat{c}) = \lambda \hat{c} \ln\left(\frac{c}{\hat{c}}\right), \quad \lambda > 0.$$

While that function has a particularly simple form, it is always increasing in the child's quality, c. Although the increase is very slow, it nevertheless means that if the parents' budget is sufficiently large, the utility from failure eventually can be larger than the utility from success, which may be unrealistic. As shown in Proposition A.2, an example of failure utility satisfying Assumption 1 that never exceeds the utility from success, provided that $v_s \geq 1$, is the following:

$$v_f(c,\hat{c}) = 1 - e^{\lambda\left(\frac{\hat{c}^2}{c} - \hat{c}\right)}, \quad \lambda > 0.$$

The two examples are illustrated in Figure 1. □

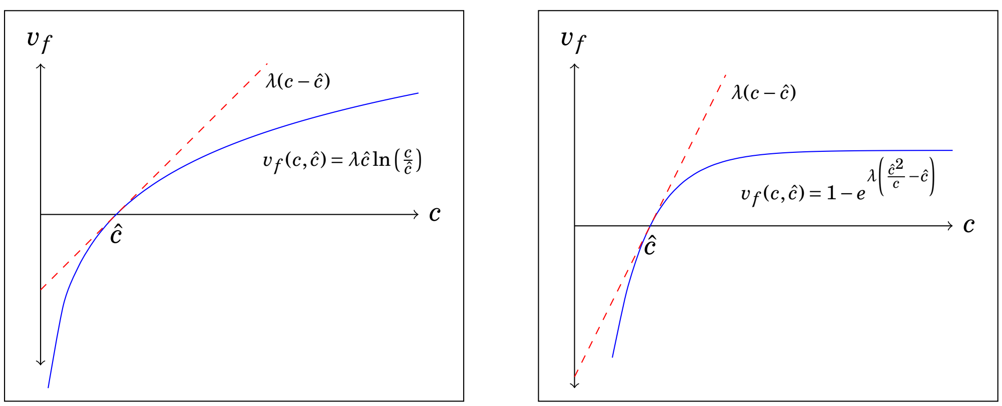

Figure 1: Two examples of failure utilities.

Because we focus on symmetric equilibria, we need to consider only incentives for unilateral deviations from profiles in which all the parents are spending the same amount. Thus, to simplify notation, we use $P(c,\hat{c})$ to denote the probability that the child of the couple spending c will succeed when everyone else is spending $\hat{c}$. That is,

$$P(c,\hat{c}) = \frac{c^r}{(N-1)\hat{c}^r + c^r}.$$

Similarly, $\pi(c,\hat{c})$ denotes the expected utility to the couple spending c when everyone else is spending $\hat{c}$:

$$\pi(c,\hat{c}) = P(c,\hat{c})v_s + (1 - P(c,\hat{c}))v_f(c,\hat{c}) - c.$$

We begin our analysis by discussing our results in relation to the Tullock model. In the standard Tullock contest wherein the failure utility always is zero, contestants do not exhaust their budgets in the equilibrium if r is not too large. More precisely, if $r \leq \frac{N}{N-1}$, the unique symmetric equilibrium is $c_{\text{sm}} = \frac{(N-1)rv_s}{N^2}$, provided that $c_{\text{sm}} \in [c_{\min}, c_{\max}]$,[13] suggesting that if λ is small so that the effect of failure is modest, then an interior equilibrium should exist in our model as well. Theorem 2.4 below verifies that that is indeed the case. However, the inclusion of failure utility means that existence requires $\lambda < \frac{N}{N-1}$, in addition to $r \leq \frac{N}{N-2}$ needed for concavity of $P(c, \hat{c})$ at $c = \hat{c}$.[14]

Theorem 2.4. *Suppose that $\lambda < \frac{N}{N-1}$ and $r \leq \frac{N}{N-2}$. Then the unique symmetric Nash equilibrium is $\boldsymbol{c}^* = (c^*, \ldots, c^*)$, where*

$$c^* = \frac{\left(\frac{r}{N}\right) v_s}{\frac{N}{N-1} - \lambda},$$

provided that $c^* \in [c_{\min},\ c_{\max}]$.

If $\lambda = 0$, $v_f(c, \hat{c})$ is approximately zero near $c = \hat{c}$, which implies that parents' behavior near symmetric spending profiles should be similar to the case in which the failure utility does not exist. Thus, it is not surprising that the equilibrium reduces to $c^* = \frac{(N-1)rv_s}{N^2}$, which is the same as in the standard model. Our required condition on r is $r \leq \frac{N}{N-2}$. In the standard Tullock contest, $r \leq \frac{N}{N-1}$ is needed to ensure that setting $c = 0$ against c_{sm} is not better than maintaining c_{sm}. Since $c_{\min} > 0$, we do not need to consider deviations to $c = 0$ in our model, and, consequently, an interior equilibrium exists for slightly larger values of r. However, $c^* > c_{\text{sm}}$ when $0 < \lambda < \frac{N}{N-1}$, so parents are spending more than the standard model even in the interior equilibrium.

As noted in the introduction, the importance of $\boldsymbol{c}^*$ being an interior equilibrium is that it is not constrained by the budget. Since parents already are spending less than their total budgets in that equilibrium, relaxing the budget constraints further, for example, because of rising incomes or cash support programs, will not increase parents' spending on their existing child. Instead, the parents can spend the additional resources on satisfying their other wants, including having more children. Unfortunately, as Theorem 2.4 makes clear, the interior equilibrium exists only if $\lambda < \frac{N}{N-1}$, since the expression for c^* given in the theorem would either be undefined or negative otherwise. In fact, the following lemma shows that when $\lambda \geq \frac{N}{N-1}$, $\pi(c, \hat{c})$ is increasing in c at $c = \hat{c}$ for all $\hat{c} > 0$.

Lemma 2.5. *Suppose that $\lambda \geq \frac{N}{N-1}$. Then for all $\hat{c} > 0$, $\frac{\partial \pi(c,\hat{c})}{\partial c}\big|_{c=\hat{c}} > 0$ and $\pi(c, \hat{c}) < \pi(\hat{c}, \hat{c})$ for all $c < \hat{c}$.*

Lemma 2.5 means that when everyone is spending $\hat{c}$, parents can obtain higher utilities by spending a little more than their peers, provided that no one else does the same. Of course, because all the parents have the same incentive, they will all end up spending the same amount again, only at a higher level. Nevertheless, such incentives to outspend one's peers exist no matter how high average spending is,

[13] See, for example, Baye, Kovenock, and de Vries (1999) or Perez-Castrillo and Verdier (1992).

[14] The conditions needed for concavity of $P(c, \hat{c})$ are given in Proposition A.3 in the Appendix.

which means that parents will ratchet up their spending until they exhaust their budgets. Therefore, the only candidate for a symmetric equilibrium spending level is $c_{\max}$. However, if everyone spends $c_{\max}$, each child's probability of success remains $\frac{1}{N}$ despite all parents exhausting their budgets. The parents clearly will be better off if everyone spends a lesser amount instead. Yet, in the absence of a coordinated de-escalation in which every couple lowers their spending simultaneously, parents must weigh their own spending decision in isolation, assuming that other parents will not match. Lemma 2.5 implies that when $\lambda \geq \frac{N}{N-1}$, reducing spending unilaterally from $c_{\max}$ lowers parents' expected utilities. Therefore, the unique symmetric equilibrium in this case is $(c_{\max}, \ldots, c_{\max})$. We state that result as Theorem 2.6.

Theorem 2.6. *Suppose that $\lambda \geq \frac{N}{N-1}$. Then $\mathbf{c_{max}} = (c_{\max}, \ldots, c_{\max})$ is the unique symmetric Nash equilibrium no matter how large $c_{\max}$ is.*

Theorem 2.6 shows that when failure has a sufficiently severe consequence, competitive pressure can drive parents to exhaust their resources on child-raising no matter how large their budgets are. Any additional resources provided to parents under such a circumstance will result only in a higher equilibrium spending level. Moreover, since $\frac{N}{N-1}$ quickly declines to 1, the value of λ for which that happens need not be extreme. Furthermore, the equilibrium is robust in that it does not depend on the parameter r, which means that budget exhaustion can occur even when the probability of success is strictly concave in one's own spending. When N is large, however, parents receive negative expected utility in that equilibrium. That is, if $N > \frac{v_s}{c_{\max}}$,

$$
\begin{aligned}
\pi(c_{\max}, c_{\max}) &= P(c_{\max}, c_{\max}) v_s + (1 - P(c_{\max}, c_{\max})) v_f(c_{\max}, c_{\max}) - c_{\max}. \\
&= \frac{v_s}{N} - c_{\max} < 0.
\end{aligned}
$$

Thus, if having children is voluntary and potential parents can anticipate that outcome, some couples may choose not to have any children. The issue is taken up in the next section, where we analyze the effect of competition on fertility.

3 Competition and fertility

We now extend the model of Section 2 by allowing parents to choose between having zero, one, or two children, in addition to choosing how much to spend on raising their children. The possibility of having two children complicates the model in that siblings typically compete only with children in their own cohort and not their sibling's. To capture that aspect, we embed our competition model in an overlapping-generations-like framework. At each period t, we assume N couples, who decide simultaneously how many children to have and how much to spend on them. If a period-t couple has two children, then their first child competes in period t, and the second child competes in period $t+1$. Thus, as illustrated in Figure 2, the first children of period-t parents are in the same cohort as the second children of the previous period's parents. Similarly, the second children of period-t parents and the first children of the next period's parents are in the same cohort.

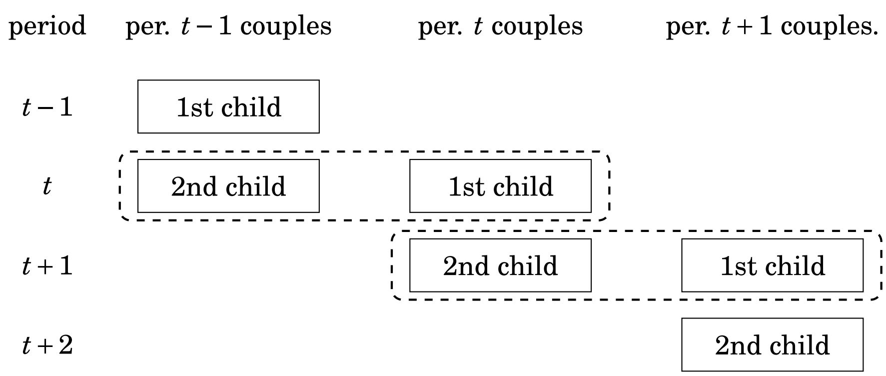

Figure 2: Dashed rectangles represent cohorts.

We assume that if a couple does not have any children, their child-raising spending and their payoff are both zero. To set a tie-breaking rule, we assume that a couple will have a child if they are indifferent between having and not having one. Similarly, when indifferent between one and two children, the couple will have two. We assume further that parents care about each of their children equally and spend the same amount on each child.[15] Thus, for any period t, a strategy of couple i who makes decision in that period takes the form $s_i = (n, c)$, where n is the number of children they will have and c is the spending level per child. Feasibility requires $c \leq \bar{c} = \frac{c_{\max}}{2}$ if the couple has two children. A period-t strategy profile is a list $\boldsymbol{s_t} = (s_1, ..., s_i, ..., s_N)$ that specifies a strategy for every couple in that period, and a strategy profile for the entire model is an infinite sequence $(\boldsymbol{s_1}, \boldsymbol{s_2}, ..., \boldsymbol{s_t}, ...)$ specifying a period strategy profile for every period.

As in Section 2, we look for a symmetric equilibrium in the model. However, since the point of our analysis is to investigate parents' fertility decisions, it would not be sensible to restrict attention from the outset to equilibria in which parents have the same number of children. Thus, we relax the notion of symmetry and require only that parents' spending levels conditioned on the number of children are symmetric. That is, we look for an equilibrium wherein parents who have the same number of children spend the same amount. A conditionally symmetric strategy profile for period t can be written more succinctly as a pair $\boldsymbol{s_t} = \big((\mathcal{N}_{1t}, c_{1t}), (\mathcal{N}_{2t}, c_{2t})\big)$, where $\mathcal{N}_{mt}$ is the set of t-period couples of having m children, and c_{mt} is their spending per child. By convention, c_{mt} is understood to be zero if $\mathcal{N}_{mt} = \emptyset$. The couples not having any children, $\mathcal{N}_{0t}$, are those who are not in $\mathcal{N}_{1t}$ or $\mathcal{N}_{2t}$. To keep the analysis tractable, we restrict attention further to stationary equilibria, such that the equilibrium strategy does not change over time. However, since no direct relationship exists for the i-th couple from different periods, we call period strategy profiles $\boldsymbol{s_t}$ and $\boldsymbol{s_{t'}}$ equivalent, written, $\boldsymbol{s_t} \equiv \boldsymbol{s_{t'}}$, if the number of couples adopting each given strategy is the same in both profiles, and we let stationarity mean $\boldsymbol{s_t} \equiv \boldsymbol{s_{t'}}$ for all t and t'.[16] Since all the

[15] We believe that it is reasonable to assume that the modal parenting behavior in developed nations does sacrifice one child for the sake of giving another child a better chance to succeed.

[16] Our reliance on stationarity requires group size N to be the same at all periods, which may not be ideal in a model of fertility. As an alternative, we may interpret stationarity as holding only locally. To be more precise, we fix a particular period t and investigate the equilibrium behavior of only the couples

period strategy profiles are equivalent in a stationary strategy profile, we represent a stationary, conditionally symmetric (SCS) strategy profile simply by its common period profile $\boldsymbol{s} = \big((\mathcal{N}_1, c_1), (\mathcal{N}_2, c_2)\big)$, denoting it without a time subscript.

To analyze a couple's incentive to deviate from a SCS profile $\hat{\boldsymbol{s}}$, fix period t and let $(s_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}})$ be the strategy profile in which period-t couple i chooses $s_i = (n, c)$ and everyone else, including the i-th couples in periods other than t, adopts the strategy specified by $\hat{\boldsymbol{s}}$. Let N_m be the number of couples having m children under $\hat{\boldsymbol{s}}$ and $1_{(i \in \mathcal{N}_m)}$ be an indicator variable that takes value 1 if period-t couple i has m children under $\hat{\boldsymbol{s}}$ and 0 otherwise. Then, if $n \geq 1$, so that couple i has at least one child under s_i, the probability that their first child will succeed is

$$P_{i1}(s_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = \frac{c^r}{N_2 c_2^r + \big(N_1 - 1_{(i \in \mathcal{N}_1)}\big) c_1^r + \big(N_2 - 1_{(i \in \mathcal{N}_2)}\big) c_2^r + c^r}.$$

Average spending on the child's competitors is

$$\hat{c}_{-i1} = \frac{N_2 c_2 + \big(N_1 - 1_{(i \in \mathcal{N}_1)}\big) c_1 + \big(N_2 - 1_{(i \in \mathcal{N}_2)}\big) c_2}{N_2 + \big(N_1 - 1_{(i \in \mathcal{N}_1)}\big) + \big(N_2 - 1_{(i \in \mathcal{N}_2)}\big)},$$

provided that the child has at least one rival. If she has no competitor, we set $\hat{c}_{-i1} = c$, so that $v_f(c, \hat{c}_{-i1}) = 0$. If $n = 2$, so that the couple has two children, the probability that their second child will succeed is

$$P_{i2}(s_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = \frac{c^r}{\big(N_2 - 1_{(i \in \mathcal{N}_2)}\big) c_2^r + N_1 c_1^r + N_2 c_2^r + c^r}.$$

Average spending on the second child's competitors is

$$\hat{c}_{-i2} = \frac{(N_2 - 1_{(i \in \mathcal{N}_2)}) c_2 + N_1 c_1 + N_2 c_2}{(N_2 - 1_{(i \in \mathcal{N}_2)}) + N_1 + N_2}$$

if the child has at least one rival, and $\hat{c}_{-i2} = c$ otherwise. The overall expected utility of couple i is

$$\pi_i(s_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = \begin{cases} 0 & \text{if } n = 0 \\ P_{i1} v_s + (1 - P_{i1}) v_f(c, \hat{c}_{-i1}) - c & \text{if } n = 1 \\ P_{i1} v_s + (1 - P_{i1}) v_f(c, \hat{c}_{-i1}) + P_{i2} v_s + (1 - P_{i2}) v_f(c, \hat{c}_{-i2}) - 2c & \text{if } n = 2. \end{cases}$$

We now are ready to define our equilibrium. (The requirement for strict inequality in the definition stems from our tie-breaking rule.)

Definition 3.1. A *stationary, conditionally symmetric (SCS) equilibrium* is a SCS strategy profile $\boldsymbol{s}^*$ such that

$$\pi_i(s_i^*, \boldsymbol{s}_{-\boldsymbol{i}}^*) \geq \pi_i(s_i, \boldsymbol{s}_{-\boldsymbol{i}}^*) \text{ for all } i \text{ and } s_i,$$

with the inequality being strict for all $s_i = (n, c)$, where n is greater than the number of children couple i has under s_i^*.

who make their decision in period t, treating the decisions of couples in periods $t-1$ and $t+1$ as fixed. Stationarity is then required only to hold between periods $t-1$, t, and $t+1$, which is more plausible because the gap between the periods presumably is short.

In the remainder of the paper, assume that $N \geq \bar{N}$, where $\bar{N}$ is the largest integer such that $\frac{v_s}{\bar{N}} \geq c_{\max}$. In the model of Section 2, where parents have exactly one child, $\bar{N}$ is the largest number of children that is consistent with parents having non-negative expected utility in equilibrium. In the current model, if $N_1 + 2N_2 < \bar{N}$, at least one couple does not have any children. At the same time, the total expenditure is $N_1 c_1 + 2N_2 c_2 < \bar{N} c_{\max} \leq v_s$, which means that a potential gain exists for the couple if they switch to having a child. Lemma 3.2 establishes that no SCS equilibrium exists in which $N_1 + 2N_2 < \bar{N}$, precisely for that reason.

Lemma 3.2. *Let $\hat{\boldsymbol{s}} = \big((\mathcal{N}_1, c_1), (\mathcal{N}_2, c_2)\big)$, where $N_1 + 2N_2 < \bar{N}$. Then $\hat{\boldsymbol{s}}$ cannot be a SCS equilibrium.*

As discussed in Section 2, Lemma 2.5 shows that $\pi(c, \hat{c})$ is increasing in c at $c = \hat{c}$ for all $\hat{c}$ in the one-child model, which implies that parents always will ratchet up their spending from any symmetric spending profile in that model. Restricting to the case of $N_1 + 2N_2 \geq \bar{N}$, Lemmas 3.3 and 3.4 below show that a similar property also holds in the current model if $r + \lambda > \frac{\bar{N}}{\bar{N}-1}$. More precisely, the proofs of the lemmas show that whenever parents choose to spend less than the maximum possible for the number of children they are having, they can obtain more utility by increasing their spending without changing the number of children they have.

Lemma 3.3. *Let $\hat{\boldsymbol{s}} = \big((\mathcal{N}_1, c_1), (\mathcal{N}_2, c_2)\big)$, where $N_1 + 2N_2 \geq \bar{N}$. Suppose that either (i) $\mathcal{N}_1 \neq \emptyset$, $\mathcal{N}_2 \neq \emptyset$, and $c_1 \leq c_2$, or (ii) $\mathcal{N}_1 \neq \emptyset$, $\mathcal{N}_2 = \emptyset$, and $c_1 < c_{\max}$. Then $\hat{\boldsymbol{s}}$ cannot be a SCS equilibrium if $r + \lambda > \frac{\bar{N}}{\bar{N}-1}$.*

Lemma 3.4. *Let $\hat{\boldsymbol{s}} = \big((\mathcal{N}_1, c_1), (\mathcal{N}_2, c_2)\big)$, where $N_1 + 2N_2 \geq \bar{N}$. Suppose that either (i) $\mathcal{N}_1 \neq \emptyset$, $\mathcal{N}_2 \neq \emptyset$, $c_2 < c_1$, and $c_2 < \bar{c}$, or (ii) $\mathcal{N}_1 = \emptyset$, $\mathcal{N}_2 \neq \emptyset$, and $c_2 < \bar{c}$. Then $\hat{\boldsymbol{s}}$ cannot be a SCS equilibrium if $r + \lambda > \frac{\bar{N}}{\bar{N}-1}$.*

The lemmas imply that the only possible candidates for an equilibrium are the ones in which couples having children spend the maximum possible, that is, SCS strategy profiles of the form $\big((\mathcal{N}_1, c_{\max}), (\emptyset, 0)\big)$, $\big((\emptyset, 0), (\mathcal{N}_2, \bar{c})\big)$ and $\big((\mathcal{N}_1, c_{\max}), (\mathcal{N}_2, \bar{c})\big)$ are possible. Whether the last two candidates turn out to be equilibria hinge on whether being forced to give up on having a second child will be a barrier to ratcheting up spending further from $\bar{c}$. Since increasing per-child spending from $\bar{c}$ to $c > \bar{c}$ brings a discrete drop in the number of children and, hence, a discontinuous change in parents' utility, the derivative-based analysis of Lemmas 3.3 and 3.4 no longer applies. Nevertheless, Lemma 3.5 below establishes that when competitive pressure is sufficiently strong, no conditionally symmetric equilibrium exists in which some parents have two children because they will prefer to concentrate their resources on raising only one child rather than dividing it between two children. As the lemma shows, the degree of competition that is needed for that to occur is not extreme. All that is required is r greater than $\ln\left(2 + \frac{2}{\bar{N}-2}\right) \Big/ \ln 2$, which is at most two and declines rapidly to one as N increases.[17]

Lemma 3.5. *Let $\hat{\boldsymbol{s}} = \big((\mathcal{N}_1, c_1), (\mathcal{N}_2, c_2)\big)$, where $N_1 + 2N_2 \geq \bar{N}$. Suppose that either (i) $\mathcal{N}_1 \neq \emptyset$, $\mathcal{N}_2 \neq \emptyset$, and $c_2 = \bar{c} < c_1$, or (ii) $\mathcal{N}_1 = \emptyset$, $\mathcal{N}_2 \neq \emptyset$, and $c_2 = \bar{c}$. Then $\hat{\boldsymbol{s}}$ cannot be*

[17] When $\bar{N} = 3$, $\frac{\ln\left(2 + \frac{2}{\bar{N}-2}\right)}{\ln(2)} = \frac{\ln(4)}{\ln(2)} = 2$. It drops to 1.17 when $\bar{N} = 10$ and continues to fall to 1 as $\bar{N} \to \infty$.

a SCS equilibrium if

$$r > \frac{\ln\left(2 + \frac{2}{\bar{N}-2}\right)}{\ln 2}.$$

Lemma 3.5 implies that if r is sufficiently large, parents are willing to forego having another child so that they can give their sole child an additional edge in the competition, leaving $\big((\mathcal{N}_1, c_{\max}), (\emptyset, 0)\big)$ as the only viable candidate for an equilibrium. Similar to the previous section, whether that constitutes an equilibrium depends on whether it will be better to lower spending unilaterally instead. The analysis is more delicate than Section 2, though, because reducing spending to a level below $\bar{c}$ potentially can give parents additional utility from a second child. Lemmas 3.6 and 3.7 show that if both the consequences of failure and the intensity of competition are sufficiently high, parents with zero children cannot increase their utility by having children, and parents having one child cannot gain more utility by reducing their spending unilaterally, even if it means gaining the ability to have two children.

Lemma 3.6. *Suppose that* $\lambda \geq \frac{\bar{N}}{\bar{N}-1}$. *Let* $\boldsymbol{s}^* = \big((\mathcal{N}_1, c_{\max}), (\emptyset, 0)\big)$, *where* $N_1 = \bar{N}$. *Let* $i \in \mathcal{N}_0$. *Then* $\pi_i(s_i^*, \boldsymbol{s}_{-\boldsymbol{i}}^*) > \pi_i(s_i, \boldsymbol{s}_{-\boldsymbol{i}}^*)$ *for all* $s_i \neq s_i^*$.

Lemma 3.7. *Suppose that* $\lambda \geq \frac{\bar{N}}{\bar{N}-1}$ *and* $r \geq \frac{\ln\left(2+\frac{1}{\bar{N}-1}\right)}{\ln 2}$. *Let* $\boldsymbol{s}^* = \big((\mathcal{N}_1, c_{\max}), (\emptyset, 0)\big)$, *where* $N_1 = \bar{N}$. *Let* $i \in \mathcal{N}_1$. *Then* $\pi_i(s_i^*, \boldsymbol{s}_{-\boldsymbol{i}}^*) \geq \pi_i(s_i, \boldsymbol{s}_{-\boldsymbol{i}}^*)$ *for all* $s_i \neq s_i^*$. *Moreover, the inequality is strict if* $s_i \neq (0, 0)$.

Together, the lemmas imply that $\bar{N}$ couples spending $c_{\max}$ on their one child is the only SCS equilibrium. We state that result as Theorem 3.8.

Theorem 3.8. *Suppose that* $\lambda \geq \frac{\bar{N}}{\bar{N}-1}$ *and* $r > \frac{\ln\left(2+\frac{2}{\bar{N}-2}\right)}{\ln 2}$. *Then* $\boldsymbol{s}^* = \big((\mathcal{N}_1, c_{\max}), (\emptyset, 0)\big)$, *where* $N_1 = \bar{N}$, *is a stationary, conditionally symmetric equilibrium no matter how large* $c_{\max}$ *is. Moreover, the equilibrium is unique up to equivalence class.*

An immediate consequence of the result is that when the parameters satisfy the theorem's hypothesis, relaxing parents' budget constraints will not have a positive effect on the equilibrium fertility rate. In fact, if the budget $c_{\max}$ is raised without a corresponding increase in v_s, the equilibrium fertility rate may even fall. That result, which is stated as Corollary 3.9, may provide an additional explanation for why a fall in fertility often is associated with rising incomes.

Corollary 3.9. *Suppose that* λ *and* r *satisfy the hypothesis of Theorem 3.8. Then the equilibrium fertility rate,* $\bar{N}/N$, *is non-decreasing in* v_s *and non-increasing in* $c_{\max}$.

4 Discussion and conclusion

The recent fertility experiences of industrialized nations raise two interesting questions. First, why East Asia, which supposedly has strong family values, has fertility rates that are among the lowest in the world. Second, why cash support programs that seem effective elsewhere have been ineffective in improving fertility rates there. The results of this paper suggest that the highly competitive educational environment in which East Asian children are raised partly may answer both questions. Our model shows that when the consequences of failure are sufficiently severe, competitive pressure can force parents to forego having additional children and concentrate their resources on increasing the quality of the one child they have. Since that result holds at all budget levels, additional cash support increases only per-child equilibrium spending and does not lead to additional children. Given that East Asia has relatively strong competition for entry into elite tertiary schools and weak social protections, it satisfies both conditions that are required for that equilibrium to emerge. Moreover, to the extent that traditional family values place tight connection between parents' utility and their children's well-being, strong family values can hinder rather than help fertility in the presence of entry-level rivalry. In our model, it is parents' intense regard for their children's future well-being that ironically makes them have fewer children.

The results highlight that raising fertility rates in highly competitive societies may be difficult because competition makes parents' fertility and spending decisions interdependent. In such environments, improving social safety nets that mitigate the consequences of failure may be more effective than programs that affect only unilateral incentives, such as maternity bonuses or education subsidies. In addition, as competitive pressures become stronger and more prevalent even outside East Asia, our results caution that nations that have so far escaped the lowest-low fertility box may experience deteriorations in their fertility rates if the strengthening competition is not accompanied by sufficient protection for those who do not succeed. More generally, our results question the notion that inequality of outcomes should be tolerated in the name of efficiency as long as equality of opportunity exists. Our model illustrates that when a large disparity between outcomes emerges, individuals may go to great lengths to avoid bad results, leading to an inefficient use of resources at the societal level. Moreover, in the presence of competition, inefficiencies can be exacerbated by equal opportunity policies. In our model, it is the very fact that everyone has an equal opportunity to achieve success that drives parents to overspend on their children's human capital accumulation and, in consequence, constrain fertility.

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A Appendix

Proposition A.1. *Suppose that the failure utility $v_f(c,\hat{c})$ satisfies Assumption 1. Then for all $\hat{c} > 0$ and $c \le \hat{c}$, $v_f(c,\hat{c}) \le \lambda(c-\hat{c})$.*

Proof. Fix $\hat{c}$ and consider any $c < \hat{c}$. By the mean value theorem, we have

$$v_f(\hat{c},\hat{c}) - v_f(c,\hat{c}) = v_f'(x,\hat{c})(\hat{c}-c) \quad \text{for some } x \in (c,\hat{c}).$$

Since $v_f'' \le 0$, v_f' is weakly decreasing, which means $v_f'(x,\hat{c}) \ge v_f'(\hat{c},\hat{c}) = \lambda$. Thus,

$$v_f(\hat{c},\hat{c}) - v_f(c,\hat{c}) \ge \lambda(\hat{c}-c).$$

Using $v_f(\hat{c},\hat{c}) = 0$, we obtain $v_f(c,\hat{c}) \le \lambda(c-\hat{c})$, as required. □

Proposition A.2. *Let $\lambda > 0$. Then $v_f(c,\hat{c}) = \lambda\hat{c}\ln\left(\frac{c}{\hat{c}}\right)$ and $v_f(c,\hat{c}) = 1 - e^{\lambda\left(\frac{\hat{c}^2}{c}-\hat{c}\right)}$ satisfy Assumption 1. Moreover, $v_f(c,\hat{c}) = 1 - e^{\lambda\left(\frac{\hat{c}^2}{c}-\hat{c}\right)} \le 1$ for all $c > 0$ and $\hat{c} > 0$.*

Proof. Here and below, let prime (′) denote a partial derivative with respect to c. First, for $v_f(c,\hat{c}) = \lambda\hat{c}\ln\left(\frac{c}{\hat{c}}\right)$, we have

$$v_f'(c,\hat{c}) = \frac{\lambda\hat{c}}{c} > 0 \quad \text{and} \quad v_f''(c,\hat{c}) = -\frac{\lambda\hat{c}}{c^2} < 0.$$

Evaluating $v_f(c,\hat{c})$ and $v_f'(c,\hat{c})$ at $c = \hat{c}$ yields

$$v_f(\hat{c},\hat{c}) = \lambda\hat{c}\ln\left(\frac{\hat{c}}{\hat{c}}\right) = 0 \quad \text{and} \quad v_f'(\hat{c},\hat{c}) = \frac{\lambda\hat{c}}{\hat{c}} = \lambda.$$

Therefore, conditions (1) and (2) of Assumption 1 are satisfied.

Second, for $v_f(c,\hat{c}) = 1 - e^{\lambda\left(\frac{\hat{c}^2}{c}-\hat{c}\right)}$, we have

$$v_f'(c,\hat{c}) = -e^{\lambda\left(\frac{\hat{c}^2}{c}-\hat{c}\right)}\left(\frac{-\lambda\hat{c}^2}{c^2}\right) = e^{\lambda\left(\frac{\hat{c}^2}{c}-\hat{c}\right)}\left(\frac{\lambda\hat{c}^2}{c^2}\right) > 0,$$

$$\text{and} \quad v_f''(c,\hat{c}) = e^{\lambda\left(\frac{\hat{c}^2}{c}-\hat{c}\right)}\left(\frac{-\lambda\hat{c}^2}{c^2}\right)\left(\frac{\lambda\hat{c}^2}{c^2}\right) + e^{\lambda\left(\frac{\hat{c}^2}{c}-\hat{c}\right)}\left(\frac{-2\lambda\hat{c}^2}{c^3}\right) = -e^{\lambda\left(\frac{\hat{c}^2}{c}-\hat{c}\right)}\left(\frac{\lambda^2\hat{c}^4}{c^4} + \frac{2\lambda\hat{c}^2}{c^3}\right) < 0.$$

Evaluating $v_f(c,\hat{c})$ and $v_f'(c,\hat{c})$ at $c = \hat{c}$ yields

$$v_f(\hat{c},\hat{c}) = 1 - e^{\lambda\left(\frac{\hat{c}^2}{\hat{c}}-\hat{c}\right)} = 1 - e^0 = 0 \quad \text{and} \quad v_f'(\hat{c},\hat{c}) = e^0(\lambda) = \lambda.$$

Therefore, conditions (1) and (2) of Assumption 1 are satisfied. Lastly, since $e^{\lambda\left(\frac{\hat{c}^2}{c}-\hat{c}\right)} > 0$ for all $c > 0$ and $\hat{c} > 0$, we have

$$v_f(c,\hat{c}) = 1 - e^{\lambda\left(\frac{\hat{c}^2}{c}-\hat{c}\right)} < 1,$$

as claimed. □

Proposition A.3. *For all $\hat{c} > 0$, $P(c,\hat{c})$ is increasing in c. It is concave at $c = \hat{c}$ if and only if $r \le \frac{N}{N-2}$. It is concave at all $c > 0$ if and only if $r \le 1$.*

Proof. Fix any $\hat{c} > 0$. Differentiating $P(c,\hat{c}) = \frac{c^r}{(N-1)\hat{c}^r + c^r}$ with respect to c yields

$$P'(c,\hat{c}) = \frac{rc^{r-1}[(N-1)\hat{c}^r + c^r] - c^r\left[rc^{r-1}\right]}{[(N-1)\hat{c}^r + c^r]^2} = \frac{r(N-1)\hat{c}^r c^{r-1} + rc^{2r-1} - rc^{2r-1}}{[(N-1)\hat{c}^r + c^r]^2}$$
$$= \frac{r(N-1)\hat{c}^r c^{r-1}}{[(N-1)\hat{c}^r + c^r]^2} > 0. \tag{1}$$

$$P''(c,\hat{c}) = \left(r(N-1)\hat{c}^r\right)\left((r-1)c^{r-2}\left[(N-1)\hat{c}^r + c^r\right]^{-2} - 2c^{r-1}\left[(N-1)\hat{c}^r + c^r\right]^{-3} rc^{r-1}\right)$$
$$= \underbrace{\left(\frac{r(N-1)\hat{c}^r c^{r-2}}{[(N-1)\hat{c}^r + c^r]^2}\right)}_{(+)}\left((r-1) - \frac{2rc^r}{(N-1)\hat{c}^r + c^r}\right). \tag{2}$$

Thus,

$$P''(c,\hat{c}) \le 0 \text{ for all } c > 0 \iff r - 1 \le \frac{2rc^r}{(N-1)\hat{c}^r + c^r} \text{ for all } c > 0$$
$$\iff r - 1 \le \inf_{c>0} \frac{2rc^r}{(N-1)\hat{c}^r + c^r} = 0 \iff r \le 1.$$

Lastly, letting $c = \hat{c}$ in expression (2), we have

$$P''(\hat{c},\hat{c}) \le 0 \iff (r-1) - \frac{2r\hat{c}^r}{(N-1)\hat{c}^r + \hat{c}^r} \le 0 \iff r - \frac{2r}{N} \le 1 \iff rN - 2r \le N$$
$$\iff r \le \frac{N}{N-2}.$$ □

Theorem 2.4. *Suppose that $\lambda < \frac{N}{N-1}$ and $r \le \frac{N}{N-2}$. Then the unique symmetric Nash equilibrium is $\mathbf{c}^* = (c^*, \ldots, c^*)$, where*

$$c^* = \frac{\left(\frac{r}{N}\right)v_s}{\frac{N}{N-1} - \lambda},$$

provided that $c^* \in [c_{\min},\ c_{\max}]$.

Proof. Fix any $\hat{c}$. Differentiating $\pi(c,\hat{c}) = P(c,\hat{c})v_s + (1 - P(c,\hat{c}))v_f(c,\hat{c}) - c$ with respect to c yields

$$\pi'(c,\hat{c}) = P'(c,\hat{c})\left(v_s - v_f(c,\hat{c})\right) + (1 - P(c,\hat{c}))v_f'(c,\hat{c}) - 1.$$

Evaluating this expression at $c = \hat{c}$ yields

$$\pi'(\hat{c},\hat{c}) = \frac{r(N-1)\hat{c}^r\hat{c}^{r-1}}{[(N-1)\hat{c}^r + \hat{c}^r]^2}\left(v_s - v_f(\hat{c},\hat{c})\right) + \left(1 - \frac{1}{N}\right)v_f'(\hat{c},\hat{c}) - 1 \quad \text{by expression (1)}$$
$$= \frac{r(N-1)\hat{c}^{2r-1}}{N^2\hat{c}^{2r}}v_s + \left(\frac{N-1}{N}\right)\lambda - 1 \quad \text{since } v_f(\hat{c},\hat{c}) = 0 \text{ and } v_f'(\hat{c},\hat{c}) = \lambda$$
$$= \frac{r(N-1)}{N^2\hat{c}}v_s + \left(\frac{N-1}{N}\right)\lambda - 1. \tag{3}$$

Thus, the first order condition for symmetric equilibrium $(c^*,\dots,c^*)$ is

$$\pi'(c^*,c^*)=0 \iff \frac{r(N-1)}{N^2c^*}v_s+\left(\frac{N-1}{N}\right)\lambda-1=0 \iff \frac{r}{Nc^*}v_s=\frac{N}{N-1}-\lambda.$$

Since the left-hand side of the last equation is positive, the equation has a solution if and only if $\lambda<\frac{N}{N-1}$. We then have

$$c^*=\frac{\left(\frac{r}{N}\right)v_s}{\frac{N}{N-1}-\lambda}.$$

Since $r\le\frac{N}{N-2}$, $P''(c^*,c^*)\le 0$ by Lemma A.3. Differentiating $\pi'(c,c^*)$ with respect to c and evaluating it at $c=c^*$ yields

$$\pi''(c^*,c^*)=\underbrace{P''(c^*,c^*)\big(v_s-v_f(c^*,c^*)\big)}_{(\le 0)(+)}-\underbrace{2P'(c^*,c^*)v_f'(c^*,c^*)}_{(+)(+)}+\underbrace{\big(1-P(c^*,c^*)\big)v_f''(c^*,c^*)}_{(+)(\le 0)}<0.$$

Therefore, $c=c^*$ is a local maximizer of $\pi(c,c^*)$ on $c>0$. Since $\pi(c,c^*)$ is a continuous function of c and has no other critical point, c^* is also the unique global maximizer. Thus, $(c^*,\dots,c^*)$ is the unique symmetric equilibrium, provided $c^*\in[c_{\min},\ c_{\max}]$. □

Lemma 2.5. *Suppose that $\lambda\ge\frac{N}{N-1}$. Then for all $\hat{c}>0$, $\frac{\partial\pi(c,\hat{c})}{\partial c}\big|_{c=\hat{c}}>0$ and $\pi(c,\hat{c})<\pi(\hat{c},\hat{c})$ for all $c<\hat{c}$.*

Proof. From expression (3), we have

$$\pi'(\hat{c},\hat{c})=\underbrace{\frac{r(N-1)}{N^2\hat{c}}v_s}_{(+)}+\left(\frac{N-1}{N}\right)\lambda-1>0 \quad \text{if } \lambda\ge\frac{N}{N-1}.$$

Next, restricting attention to $c<\hat{c}$, we have

$$\begin{aligned}
&\pi(c,\hat{c})<\pi(\hat{c},\hat{c})\\
\iff\ & P(c,\hat{c})v_s+(1-P(c,\hat{c}))v_f(c,\hat{c})-c<P(\hat{c},\hat{c})v_s+(1-P(\hat{c},\hat{c}))v_f(\hat{c},\hat{c})-\hat{c}\\
\iff\ & (1-P(c,\hat{c}))v_f(c,\hat{c})+(\hat{c}-c)<\left(\frac{1}{N}-\frac{c^r}{(N-1)\hat{c}^r+c^r}\right)v_s. \qquad (4)
\end{aligned}$$

Since $c<\hat{c}$, the right-hand side of inequality (4) is positive. Using Proposition A.1, we obtain

$$\begin{aligned}
v_f(c,\hat{c})\le\lambda(c-\hat{c}) &\iff \frac{v_f(c,\hat{c})}{c-\hat{c}}\ge\lambda \quad \text{since } c-\hat{c}<0\\
&\implies \frac{v_f(c,\hat{c})}{c-\hat{c}}\ge\frac{N}{N-1}>\frac{(N-1)\hat{c}^r+c^r}{(N-1)\hat{c}^r}=\frac{1}{1-P(c,\hat{c})}\\
&\implies (1-P(c,\hat{c}))v_f(c,\hat{c})<c-\hat{c}\\
&\iff (1-P(c,\hat{c}))v_f(c,\hat{c})+(\hat{c}-c)<0.
\end{aligned}$$

Therefore, the left-hand side of inequality (4) is negative. Thus, the inequality is satisfied, and $\pi(c,\hat{c})<\pi(\hat{c},\hat{c})$ follows. □

Theorem 2.6. *Suppose that* $\lambda \geq \frac{N}{N-1}$. *Then* $\boldsymbol{c}_{\mathbf{max}} = (c_{\max}, \ldots, c_{\max})$ *is the unique symmetric Nash equilibrium no matter how large* $c_{\max}$ *is.*

Proof. Given in the text. □

Lemma 3.2. *Let* $\hat{\boldsymbol{s}} = \big((\mathcal{N}_1, c_1), (\mathcal{N}_2, c_2)\big)$, *where* $N_1 + 2N_2 < \bar{N}$. *Then* $\hat{\boldsymbol{s}}$ *cannot be a SCS equilibrium.*

Proof. Let $\hat{\boldsymbol{s}}$ be a SCS strategy profile such that $N_1 + 2N_2 < \bar{N}$. Then $N_1 + N_2 \leq N_1 + 2N_2 < \bar{N} \leq N$, so $\mathcal{N}_0 \neq \emptyset$. Choose $i \in \mathcal{N}_0$. Then $\hat{s}_i = (0,0)$ and $\pi_i(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = 0$. Suppose couple i switches to strategy $s_i = (1, c_{\max})$. Then

$$\hat{c}_{-i1} = \begin{cases} \frac{N_1 c_1 + 2N_2 c_2}{N_1 + 2N_2} \leq c_{\max} & \text{if } n_j \neq 0 \text{ for some } j \neq i \\ c_{\max} & \text{if } n_j = 0 \text{ for all } j \neq i. \end{cases}$$

In either case, we have $\hat{c}_{-i1} \leq c_{\max}$, which means $v_f(c_{\max}, \hat{c}_{-i1}) \geq 0$. Thus,

$$\begin{aligned} \pi_i(s_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) &= P_{i1}(s_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) v_s + (1 - P_{i1}(s_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}})) v_f(c_{\max}, \hat{c}_{-i1}) - c_{\max} \\ &\geq P_{i1}(s_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) v_s - c_{\max} = \left(\frac{c_{\max}^r}{N_1 c_1^r + 2N_2 c_2^r + c_{\max}^r} \right) v_s - c_{\max} \\ &\geq \frac{1}{N_1 + 2N_2 + 1} v_s - c_{\max} \geq \frac{1}{\bar{N}} v_s - c_{\max} \geq 0 = \pi_i(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}). \end{aligned}$$

If $\pi_i(s_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) > \pi_i(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}})$, couple i will prefer to switch to s_i. If $\pi_i(s_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = \pi_i(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}})$, the tie-breaking rule means couple i will switch in this case as well. Therefore, $\hat{\boldsymbol{s}}$ cannot be an equilibrium. □

Lemma 3.3. *Let* $\hat{\boldsymbol{s}} = \big((\mathcal{N}_1, c_1), (\mathcal{N}_2, c_2)\big)$, *where* $N_1 + 2N_2 \geq \bar{N}$. *Suppose that either (i)* $\mathcal{N}_1 \neq \emptyset$, $\mathcal{N}_2 \neq \emptyset$, *and* $c_1 \leq c_2$, *or (ii)* $\mathcal{N}_1 \neq \emptyset$, $\mathcal{N}_2 = \emptyset$, *and* $c_1 < c_{\max}$. *Then* $\hat{\boldsymbol{s}}$ *cannot be a SCS equilibrium if* $r + \lambda > \frac{\bar{N}}{\bar{N}-1}$.

Proof. First, note that $N_1 + 2N_2 \geq \bar{N}$, together with $c_1 \leq c_2$ or $N_2 = 0$, means

$$r + \lambda > \frac{\bar{N}}{\bar{N} - 1} \geq \frac{N_1 + 2N_2}{N_1 + 2N_2 - 1} = \frac{N_1 c_1^r + 2N_2 c_1^r}{(N_1 - 1) c_1^r + 2N_2 c_1^r} \geq \frac{N_1 c_1^r + 2N_2 c_2^r}{(N_1 - 1) c_1^r + 2N_2 c_2^r}. \tag{5}$$

Since $\mathcal{N}_1 \neq \emptyset$ in both case (i) and case (ii), choose $i \in \mathcal{N}_1$. Then $\hat{s}_i = (1, c_1)$. If $\pi_i(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) < 0$, not having a child and obtaining zero utility is strictly better than $\hat{s}_i$, which means $\hat{\boldsymbol{s}}$ is not an equilibrium. Thus, we may assume $\pi_i(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) \geq 0$ for the remainder of the proof. Since $c_1 \leq c_2$ or $N_2 = 0$,

$$\hat{c}_{-i1} = \frac{(N_1 - 1) c_1 + 2N_2 c_2}{N_1 + 2N_2 - 1} \geq c_1,$$

which means $v_f(c_1, \hat{c}_{-i1}) \leq 0$. Thus,

$$\begin{aligned} &\pi_i(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = P_{i1}(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) v_s + (1 - P_{i1}(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}})) v_f(c_1, \hat{c}_{-i1}) - c_1 \geq 0 \\ \Longrightarrow\ & P_{i1}(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) v_s - c_1 = \frac{c_1^r}{N_1 c_1^r + 2N_2 c_2^r} v_s - c_1 \geq 0 \iff v_s \geq \frac{N_1 c_1^r + 2N_2 c_2^r}{c_1^{r-1}}. \end{aligned} \tag{6}$$

For any $s_i = (1, c)$,

$$\pi_i((1,c), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = P_{i1}((1,c), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) v_s + (1 - P_{i1}((1,c), \hat{\boldsymbol{s}}_{-\boldsymbol{i}})) v_f(c, \hat{c}_{-i1}) - c$$

$$\text{where } P_{i1}((1,c), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = \frac{c^r}{(N_1 - 1)c_1^r + 2N_2 c_2^r + c^r}.$$

Here and below, let prime ($'$) denote a partial derivative with respect to c. Then

$$\pi_i'((1,c), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = P_{i1}'((1,c), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) \left(v_s - v_f(c, \hat{c}_{-i1})\right) + (1 - P_{i1}((1,c), \hat{\boldsymbol{s}}_{-\boldsymbol{i}})) v_f'(c, \hat{c}_{-i1}) - 1,$$

$$P_{i1}'((1,c), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = \frac{rc^{r-1}\left((N_1 - 1)c_1^r + 2N_2 c_2^r + c^r\right) - c^r rc^{r-1}}{\left[(N_1 - 1)c_1^r + 2N_2 c_2^r + c^r\right]^2} = \frac{rc^{r-1}\left((N_1 - 1)c_1^r + 2N_2 c_2^r\right)}{\left[(N_1 - 1)c_1^r + 2N_2 c_2^r + c^r\right]^2}.$$

Evaluating the partial derivatives at $c = c_1$ and using inequality (6) yield

$$\begin{aligned}
\pi_i'((1,c_1), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) &= P_{i1}'((1,c_1), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) \big(v_s - \underbrace{v_f(c_1, \hat{c}_{-i1})}_{(\leq 0)}\big) + (1 - P_{i1}((1,c_1), \hat{\boldsymbol{s}}_{-\boldsymbol{i}})) \underbrace{v_f'(c_1, \hat{c}_{-i1})}_{(\geq \lambda \text{ since } v_f'' \leq 0)} - 1 \\
&\geq P_{i1}'((1,c_1), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) v_s + (1 - P_{i1}((1,c_1), \hat{\boldsymbol{s}}_{-\boldsymbol{i}})) \lambda - 1 \\
&= \left(\frac{rc_1^{r-1}\left((N_1 - 1)c_1^r + 2N_2 c_2^r\right)}{\left[N_1 c_1^r + 2N_2 c_2^r\right]^2}\right) v_s + \left(\frac{(N_1 - 1)c_1^r + 2N_2 c_2^r}{N_1 c_1^r + 2N_2 c_2^r}\right) \lambda - 1 \\
&\geq \left(\frac{rc_1^{r-1}\left((N_1 - 1)c_1^r + 2N_2 c_2^r\right)}{\left[N_1 c_1^r + 2N_2 c_2^r\right]^2}\right) \left(\frac{N_1 c_1^r + 2N_2 c_2^r}{c_1^{r-1}}\right) + \frac{\lambda\left((N_1 - 1)c_1^r + 2N_2 c_2^r\right)}{N_1 c_1^r + 2N_2 c_2^r} - 1 \\
&= \frac{r\left((N_1 - 1)c_1^r + 2N_2 c_2^r\right)}{N_1 c_1^r + 2N_2 c_2^r} + \frac{\lambda\left((N_1 - 1)c_1^r + 2N_2 c_2^r\right)}{N_1 c_1^r + 2N_2 c_2^r} - 1 \\
&= \frac{(r + \lambda)\left((N_1 - 1)c_1^r + 2N_2 c_2^r\right)}{N_1 c_1^r + 2N_2 c_2^r} - 1 \\
&> 0 \text{ by inequality (5).}
\end{aligned}$$

This implies that couple i will prefer to increase their spending from c_1, which is feasible since $c_1 < c_{\max}$. Therefore, $\hat{\boldsymbol{s}} = ((1, c_1), \hat{\boldsymbol{s}}_{-\boldsymbol{i}})$ cannot be an equilibrium. □

Lemma 3.4. *Let $\hat{\boldsymbol{s}} = \left((\mathcal{N}_1, c_1), (\mathcal{N}_2, c_2)\right)$, where $N_1 + 2N_2 \geq \bar{N}$. Suppose that either (i) $\mathcal{N}_1 \neq \emptyset$, $\mathcal{N}_2 \neq \emptyset$, $c_2 < c_1$, and $c_2 < \bar{c}$, or (ii) $\mathcal{N}_1 = \emptyset$, $\mathcal{N}_2 \neq \emptyset$, and $c_2 < \bar{c}$. Then $\hat{\boldsymbol{s}}$ cannot be a SCS equilibrium if $r + \lambda > \frac{\bar{N}}{\bar{N} - 1}$.*

Proof. Similar to the proof of Lemma 3.3, $c_2 < c_1$ or $N_1 = 0$ means

$$r + \lambda > \frac{\bar{N}}{\bar{N} - 1} \geq \frac{N_1 + 2N_2}{N_1 + 2N_2 - 1} = \frac{N_1 c_2^r + 2N_2 c_2^r}{N_1 c_2^r + (2N_2 - 1)c_2^r} \geq \frac{N_1 c_1^r + 2N_2 c_2^r}{N_1 c_1^r + (2N_2 - 1)c_2^r}. \tag{7}$$

Next, since $\mathcal{N}_2 \neq \emptyset$ in both case (i) and case (ii), choose $i \in \mathcal{N}_2$. Then $\hat{s}_i = (2, c_2)$. As in the proof of Lemma 3.3, we can assume that $\pi(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) \geq 0$. Since $c_2 < c_1$ or $N_1 = 0$,

$$\hat{c}_{-i1} = \hat{c}_{-i2} = \frac{N_1 c_1 + (2N_2 - 1)c_2}{N_1 + 2N_2 - 1} \geq c_2,$$

which means $v_f(c_2, \hat{c}_{-i1}) = v_f(c_2, \hat{c}_{-i2}) \leq 0$. In addition,

$$P_{i1}(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = P_{i2}(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = \frac{c_2^r}{N_1 c_1^r + 2N_2 c_2^r}.$$

Thus,

$$\pi_i(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = 2\left(P_{i1}(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}})v_s + (1 - P_{i1}(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}))v_f(c_2, \hat{c}_{-i1}) - c_2\right) \geq 0$$

$$\implies 2(P_{i1}(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}})v_s - c_2) = 2\left(\frac{c_2^r}{N_1 c_1^r + 2N_2 c_2^r} v_s - c_2\right) \geq 0 \iff v_s \geq \frac{N_1 c_1^r + 2N_2 c_2^r}{c_2^{r-1}}. \quad (8)$$

For any $s_i = (2, c)$,

$$P_{i1}((2,c), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = P_{i2}((2,c), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = \frac{c^r}{N_1 c_1^r + (2N_2 - 1)c_2^r + c^r},$$

which implies

$$\pi_i((2,c), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = 2\left(P_{i1}((2,c), \hat{\boldsymbol{s}}_{-\boldsymbol{i}})v_s + (1 - P_{i1}((2,c), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}))v_f(c, \hat{c}_{-i1}) - c\right).$$

Thus,

$$\pi_i'((2,c), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = 2\left(P_{i1}'((2,c), \hat{\boldsymbol{s}}_{-\boldsymbol{i}})\left(v_s - v_f(c, \hat{c}_{-i1})\right) + (1 - P_{i1}((2,c), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}))v_f'(c, \hat{c}_{-i1}) - 1\right)$$

$$P_{i1}'((2,c), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = \frac{rc^{r-1}\left(N_1 c_1^r + (2N_2 - 1)c_2^r + c^r\right) - c^r rc^{r-1}}{\left[N_1 c_1^r + (2N_2 - 1)c_2^r + c^r\right]^2} = \frac{rc^{r-1}\left(N_1 c_1^r + (2N_2 - 1)c_2^r\right)}{\left[N_1 c_1^r + (2N_2 - 1)c_2^r + c^r\right]^2}.$$

Evaluating the partial derivatives at $c = c_2$ and using inequality (8) yield

$$\begin{aligned}
\pi_i'((2,c_2), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) &= 2\left(P_{i1}'((2,c_2), \hat{\boldsymbol{s}}_{-\boldsymbol{i}})\big(v_s - \underbrace{v_f(c_2, \hat{c}_{-i1})}_{(\leq 0)}\big) + (1 - P_{i1}((2,c_2), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}))\underbrace{v_f'(c_2, \hat{c}_{-i1})}_{(\geq \lambda)} - 1\right) \\
&\geq 2\left(P_{i1}'((2,c_2), \hat{\boldsymbol{s}}_{-\boldsymbol{i}})v_s + (1 - P_{i1}((2,c_2), \hat{\boldsymbol{s}}_{-\boldsymbol{i}}))\lambda - 1\right) \\
&\geq 2\left(\left(\frac{rc_2^{r-1}\left(N_1 c_1^r + (2N_2 - 1)c_2^r\right)}{\left[N_1 c_1^r + 2N_2 c_2^r\right]^2}\right)\left(\frac{N_1 c_1^r + 2N_2 c_2^r}{c_2^{r-1}}\right) + \frac{\lambda\left(N_1 c_1^r + (2N_2 - 1)c_2^r\right)}{N_1 c_1^r + 2N_2 c_2^r} - 1\right) \\
&= 2\left(\frac{r\left(N_1 c_1^r + (2N_2 - 1)c_2^r\right)}{N_1 c_1^r + 2N_2 c_2^r} + \frac{\lambda\left(N_1 c_1^r + (2N_2 - 1)c_2^r\right)}{N_1 c_1^r + 2N_2 c_2^r} - 1\right) \\
&= 2\left(\frac{(r + \lambda)\left(N_1 c_1^r + (2N_2 - 1)c_2^r\right)}{N_1 c_1^r + 2N_2 c_2^r} - 1\right) \\
&> 0 \text{ by inequality (7).}
\end{aligned}$$

This implies that couple i will prefer to increase their spending from c_2, which is feasible since $c_2 < \bar{c}$. Therefore, $\hat{\boldsymbol{s}} = ((2, c_2), \hat{\boldsymbol{s}}_{-\boldsymbol{i}})$ cannot be an equilibrium. ☐

Lemma 3.5. *Let $\hat{\boldsymbol{s}} = \left((\mathcal{N}_1, c_1), (\mathcal{N}_2, c_2)\right)$, where $N_1 + 2N_2 \geq \bar{N}$. Suppose that either (i) $\mathcal{N}_1 \neq \emptyset$, $\mathcal{N}_2 \neq \emptyset$, and $c_2 = \bar{c} < c_1$, or (ii) $\mathcal{N}_1 = \emptyset$, $\mathcal{N}_2 \neq \emptyset$, and $c_2 = \bar{c}$. Then $\hat{\boldsymbol{s}}$ cannot be a SCS equilibrium if*

$$r > \frac{\ln\left(2 + \frac{2}{\bar{N} - 2}\right)}{\ln 2}.$$

Proof. Since $\mathcal{N}_2 \neq \emptyset$ in both case (i) and case (ii), choose $i \in \mathcal{N}_2$. Then $\hat{s}_i = (2, \bar{c})$. Since $c_2 = \bar{c} < c_1$ or $N_1 = 0$,

$$\hat{c}_{-i1} = \hat{c}_{-i2} = \frac{N_1 c_1 + (2N_2 - 1)\bar{c}}{N_1 + 2N_2 - 1} \geq \bar{c},$$

which means $v_f(\bar{c}, \hat{c}_{-i1}) = v_f(\bar{c}, \hat{c}_{-i2}) \leq 0$. As in the proof of Lemma 3.4, $P_{i1}(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) = P_{i2}(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}})$. Thus,

$$\begin{aligned}\pi_i(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) &= 2\left(P_{i1}(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}})v_s + (1 - P_{i1}(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}))v_f(\bar{c}, \hat{c}_{-i1}) - \bar{c}\right) \\ &\leq 2(P_{i1}(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}})v_s - \bar{c}) = \left(\frac{2\bar{c}^r}{N_1 c_1^r + 2N_2 \bar{c}^r}\right)v_s - 2\bar{c}. \qquad (9)\end{aligned}$$

Let $s_i = (1, c_{\max}) = (1, 2\bar{c})$. Then since $v_f(c_{\max}, \hat{c}_{-i1}) > 0$,

$$\begin{aligned}\pi_i(s_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) &= P_{i1}(s_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}})v_s + (1 - P_{i1}(s_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}))v_f(c_{\max}, \hat{c}_{-i1}) - c_{\max} \\ &> P_{i1}(s_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}})v_s - c_{\max} = \left(\frac{(2\bar{c})^r}{N_1 c_1^r + (2N_2 - 1)\bar{c}^r + (2\bar{c})^r}\right)v_s - c_{\max}. \qquad (10)\end{aligned}$$

Next, using $N_1 + 2N_2 \geq \bar{N}$ yields

$$\begin{aligned} r > \frac{\ln\left(2 + \frac{2}{\bar{N}-2}\right)}{\ln 2} &\implies 2^r > 2 + \frac{2}{\bar{N} - 2} \geq 2 + \frac{2}{N_1 + 2N_2 - 2} = \frac{2N_1 + 4N_2 - 2}{N_1 + 2N_2 - 2} \\ &\iff 2^r(N_1 + 2N_2 - 2) > 2N_1 + 4N_2 - 2 \\ &\iff 2^r N_1 - 2N_1 > 4N_2 - 2 + 2^r 2 - 2^r 2N_2. \qquad (11)\end{aligned}$$

Suppose that we are in case (i) of the hypothesis. Then using $c_1 > \bar{c}$, we obtain

$$\begin{aligned}\text{inequality (11)} &\implies (2^r N_1 - 2N_1)\bar{c}^r c_1^r > (4N_2 - 2 + 2^r 2 - 2^r 2N_2)\bar{c}^r \bar{c}^r \\ &\iff 2^r \bar{c}^r N_1 c_1^r + 2^r \bar{c}^r 2N_2 \bar{c}^r > 2\bar{c}^r N_1 c_1^r + 2\bar{c}^r(2N_2 - 1)\bar{c}^r + 2\bar{c}^r 2^r \bar{c}^r \\ &\iff \left(\frac{(2\bar{c})^r}{N_1 c_1^r + (2N_2 - 1)\bar{c}^r + (2\bar{c})^r}\right)v_s - c_{\max} > \left(\frac{2\bar{c}^r}{N_1 c_1^r + 2N_2 \bar{c}^r}\right)v_s - 2\bar{c} \\ &\implies \pi_i(s_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) > \pi_i(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) \quad \text{by inequalities (9) and (10).}\end{aligned}$$

If we are in case (ii), then using $N_1 = 0$, we obtain

$$\begin{aligned}\text{inequality (11)} &\implies 0 > 4N_2 - 2 + 2^r 2 - 2^r 2N_2 \\ &\iff 2^r N_2 > 2N_2 - 1 + 2^r \\ &\iff \left(\frac{(2\bar{c})^r}{(2N_2 - 1)\bar{c}^r + (2\bar{c})^r}\right)v_s - c_{\max} > \left(\frac{1}{N_2}\right)v_s - 2\bar{c} \\ &\implies \pi_i(s_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) > \pi_i(\hat{s}_i, \hat{\boldsymbol{s}}_{-\boldsymbol{i}}) \quad \text{by inequalities (9) and (10).}\end{aligned}$$

Therefore, $\hat{\boldsymbol{s}}$ cannot be an equilibrium. □

Lemma 3.6. *Suppose that* $\lambda \geq \frac{\bar{N}}{\bar{N}-1}$. *Let* $\boldsymbol{s}^* = \left((\mathcal{N}_1, c_{\max}), (\emptyset, 0)\right)$, *where* $N_1 = \bar{N}$. *Let* $i \in \mathcal{N}_0$. *Then* $\pi_i(s_i^*, \boldsymbol{s}^*_{-\boldsymbol{i}}) > \pi_i(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})$ *for all* $s_i \neq s_i^*$.

Proof. Since the lemma holds vacuously if $\mathcal{N}_0 = \emptyset$, assume that $\mathcal{N}_0 \neq \emptyset$, and let $i \in \mathcal{N}_0$. Then $s_i^* = (0,0)$ and $\pi_i(s_i^*, \boldsymbol{s}_{-\boldsymbol{i}}^*) = 0$. Consider any $s_i = (n,c) \neq s_i^*$. Then $n = 1$ or 2. Suppose $n = 1$. Since $c \leq c_{\max}$, we have

$$P_{i1}(s_i, \boldsymbol{s}_{-\boldsymbol{i}}^*) = \frac{c^r}{N_1 c_{\max}^r + c^r} \quad \text{and} \quad \hat{c}_{-i1} = \frac{N_1 c_{\max}}{N_1} = c_{\max}.$$

Proposition A.1 and $c \leq c_{\max}$ imply

$$\begin{aligned}
v_f(c, c_{\max}) &\leq \lambda(c - c_{\max}) \leq \left(\frac{\bar{N}}{\bar{N}-1}\right)(c - c_{\max}) \leq \left(\frac{N_1+1}{N_1}\right)(c - c_{\max}) \\
&\leq \left(\frac{N_1 c_{\max}^r + c^r}{N_1 c_{\max}^r}\right)(c - c_{\max}) \\
\Longrightarrow &\left(\frac{N_1 c_{\max}^r}{N_1 c_{\max}^r + c^r}\right) v_f(c, c_{\max}) + (c_{\max} - c) \leq 0. \qquad (12)
\end{aligned}$$

By the definition of $\bar{N}$, we have $c_{\max} > \frac{v_s}{\bar{N}+1} = \frac{v_s}{N_1+1}$. Therefore,

$$\begin{aligned}
\pi_i(s_i, \boldsymbol{s}_{-\boldsymbol{i}}^*) &= P_{i1}(s_i, \boldsymbol{s}_{-\boldsymbol{i}}^*) v_s + \left(1 - P_{i1}(s_i, \boldsymbol{s}_{-\boldsymbol{i}}^*)\right) v_f(c, \hat{c}_{-i1}) - c \\
&= \frac{c^r}{N_1 c_{\max}^r + c^r} v_s + \left(\frac{N_1 c_{\max}^r}{N_1 c_{\max}^r + c^r}\right) v_f(c, c_{\max}) - c \\
&\leq \frac{c_{\max}^r}{N_1 c_{\max}^r + c_{\max}^r} v_s + \left(\frac{N_1 c_{\max}^r}{N_1 c_{\max}^r + c^r}\right) v_f(c, c_{\max}) - c \\
&= \frac{v_s}{N_1+1} + \left(\frac{N_1 c_{\max}^r}{N_1 c_{\max}^r + c^r}\right) v_f(c, c_{\max}) - c \\
&< \left(\frac{N_1 c_{\max}^r}{N_1 c_{\max}^r + c^r}\right) v_f(c, c_{\max}) + (c_{\max} - c) \\
&\leq 0 = \pi_i(s_i^*, \boldsymbol{s}_{-\boldsymbol{i}}^*) \quad \text{by inequality (12).}
\end{aligned}$$

Next, suppose $n = 2$. Then $c \leq \bar{c}$, and we have

$$P_{i1}(s_i, \boldsymbol{s}_{-\boldsymbol{i}}^*) = P_{i2}(s_i, \boldsymbol{s}_{-\boldsymbol{i}}^*) = \frac{c^r}{N_1 c_{\max}^r + c^r} \quad \text{and} \quad \hat{c}_{-i1} = \hat{c}_{-i2} = \frac{N_1 c_{\max}}{N_1} = c_{\max}$$

Thus, using the same reasoning as $n = 1$ case, we obtain

$$\begin{aligned}
\pi_i(s_i, \boldsymbol{s}_{-\boldsymbol{i}}^*) &= 2\left(P_{i1}(s_i, \boldsymbol{s}_{-\boldsymbol{i}}^*) v_s + \left(1 - P_{i1}(s_i, \boldsymbol{s}_{-\boldsymbol{i}}^*)\right) v_f(c, c_{\max}) - c\right) \\
&= 2\left(\left(\frac{c^r}{N_1 c_{\max}^r + c^r}\right) v_s + \left(\frac{N_1 c_{\max}^r}{N_1 c_{\max}^r + c^r}\right) v_f(c, c_{\max}) - c\right) < 0 = \pi(s_i^*, \boldsymbol{s}_{-\boldsymbol{i}}^*).
\end{aligned}$$

Therefore, we have $\pi_i(s_i^*, \boldsymbol{s}_{-\boldsymbol{i}}^*) > \pi(s_i, \boldsymbol{s}_{-\boldsymbol{i}}^*)$ in both cases. □

Lemma 3.7. *Suppose that* $\lambda \geq \frac{\bar{N}}{\bar{N}-1}$ *and* $r \geq \frac{\ln\left(2+\frac{1}{\bar{N}-1}\right)}{\ln 2}$. *Let* $\boldsymbol{s}^* = \left((\mathcal{N}_1, c_{\max}), (\emptyset, 0)\right)$, *where* $N_1 = \bar{N}$. *Let* $i \in \mathcal{N}_1$. *Then* $\pi_i(s_i^*, \boldsymbol{s}_{-\boldsymbol{i}}^*) \geq \pi_i(s_i, \boldsymbol{s}_{-\boldsymbol{i}}^*)$ *for all* $s_i \neq s_i^*$. *Moreover, the inequality is strict if* $s_i \neq (0,0)$.

Proof. Let $i \in \mathcal{N}_1$. Then $s_i^* = (1, c_{\max})$ and $\pi_i(s_i^*, \boldsymbol{s}_{-\boldsymbol{i}}^*) = \frac{v_s}{N_1} - c_{\max}$. Consider any $s_i = (n,c) \neq s_i^*$. Suppose $n = 0$. Then $N_1 = \bar{N}$ implies $\pi_i(s_i^*, \boldsymbol{s}_{-\boldsymbol{i}}^*) = \frac{v_s}{\bar{N}} - c_{\max} \geq 0 = \pi_i(s_i, \boldsymbol{s}_{-\boldsymbol{i}}^*)$.

Next, suppose $n = 1$. Since $s_i \neq s_i^*$, we have $c < c_{\max}$. We also have

$$P_{i1}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}}) = \frac{c^r}{(N_1-1)c^r_{\max} + c^r} \quad \text{and} \quad \hat{c}_{-i1} = \frac{(N_1-1)c_{\max}}{N_1-1} = c_{\max}.$$

Thus,

$$\begin{aligned}
&\pi_i(s_i^*, \boldsymbol{s}^*_{-\boldsymbol{i}}) > \pi_i(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}}) \\
\iff &\frac{1}{N_1}v_s - c_{\max} > P_{i1}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})v_s + \left(1 - P_{i1}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})\right) v_f(c, \hat{c}_{-i1}) - c \\
\iff &\left(\frac{1}{N_1} - \frac{c^r}{(N_1-1)c^r_{\max} + c^r}\right) v_s > \left(1 - P_{i1}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})\right) v_f(c, c_{\max}) + (c_{\max} - c). \qquad (13)
\end{aligned}$$

Since $c < c_{\max}$, the left-hand side of inequality (13) is positive. In contrast, the right-hand side is negative because $c < c_{\max}$ and Proposition A.1 yield

$$\begin{aligned}
v_f(c, c_{\max}) &\leq \lambda(c - c_{\max}) \leq \left(\frac{\bar{N}}{\bar{N}-1}\right)(c - c_{\max}) = \left(\frac{N_1}{N_1-1}\right)(c - c_{\max}) \\
&< \left(\frac{(N_1-1)c^r_{\max} + c^r}{(N_1-1)c^r_{\max}}\right)(c - c_{\max}) = \left(\frac{1}{1 - P_{i1}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})}\right)(c - c_{\max}) \\
&\implies \left(1 - P_{i1}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})\right) v_f(c, c_{\max}) + (c_{\max} - c) < 0.
\end{aligned}$$

Therefore, inequality (13) is satisfied, which means $\pi_i(s_i^*, \boldsymbol{s}^*_{-\boldsymbol{i}}) > \pi_i(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})$.

Finally, suppose $n = 2$. Then $c \leq \bar{c} < c_{\max}$. We have

$$\begin{aligned}
P_{i1}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}}) &= \frac{c^r}{(N_1-1)c^r_{\max} + c^r} \quad \text{and} \quad P_{i2}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}}) = \frac{c^r}{N_1 c^r_{\max} + c^r} < P_{i1}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}}) \\
\hat{c}_{-i1} &= \frac{(N_1-1)c_{\max}}{N_1-1} = c_{\max} \quad \text{and} \quad \hat{c}_{-i2} = \frac{N_1 c_{\max}}{N_1} = c_{\max}.
\end{aligned}$$

Since $v_f(c, c_{\max}) < 0$,

$$\begin{aligned}
\pi_i(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}}) = &P_{i1}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})v_s + \left(1 - P_{i1}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})\right) v_f(c, \hat{c}_{-i1}) \\
&+ P_{i2}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})v_s + \left(1 - P_{i2}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})\right) v_f(c, \hat{c}_{-i2}) - 2c \\
< &2P_{i1}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})v_s + 2\left(1 - P_{i1}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})\right) v_f(c, c_{\max}) - 2c.
\end{aligned}$$

Since

$$\begin{aligned}
&\left(\frac{1}{N_1} - \frac{2c^r}{(N_1-1)c^r_{\max} + c^r}\right) v_s + c_{\max} > 2\left(1 - P_{i1}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})\right) v_f(c, c_{\max}) + 2(c_{\max} - c) \qquad (14) \\
&\iff \frac{1}{N_1}v_s - c_{\max} > 2P_{i1}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})v_s + 2\left(1 - P_{i1}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})\right) v_f(c, c_{\max}) - 2c \\
&\implies \pi_i(s_i^*, \boldsymbol{s}^*_{-\boldsymbol{i}}) > \pi_i(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}}),
\end{aligned}$$

it is enough to show inequality (14). The left-hand side of inequality (14) is positive. To see this, note that

$$r \geq \frac{\ln\left(2 + \frac{1}{\bar{N}-1}\right)}{\ln 2} \iff 2^r \geq 2 + \frac{1}{\bar{N}-1} = 2 + \frac{1}{N_1-1} = \frac{2N_1-1}{N_1-1}$$

$$\iff (N_1-1)2^r+1 \geq 2N_1$$
$$\iff (N_1-1)c_{\max}^r + \left(\frac{c_{\max}}{2}\right)^r \geq 2N_1\left(\frac{c_{\max}}{2}\right)^r$$
$$\iff \frac{1}{N_1} - \frac{2\left(\frac{c_{\max}}{2}\right)^r}{(N_1-1)c_{\max}^r + \left(\frac{c_{\max}}{2}\right)^r} \geq 0.$$

Since $c \leq \bar{c} = \frac{c_{\max}}{2}$, this in turn implies

$$\left(\frac{1}{N_1} - \frac{2c^r}{(N_1-1)c_{\max}^r + c^r}\right)v_s + c_{\max} \geq \left(\frac{1}{N_1} - \frac{2\left(\frac{c_{\max}}{2}\right)^r}{(N_1-1)c_{\max}^r + \left(\frac{c_{\max}}{2}\right)^r}\right)v_s + c_{\max} > 0,$$

as required. In contrast, the right-hand side of inequality (14) is negative. To see this, we appeal to Proposition A.1 again:

$$\begin{aligned}
v_f(c, c_{\max}) &\leq \lambda(c - c_{\max}) \leq \left(\frac{\bar{N}}{\bar{N}-1}\right)(c - c_{\max}) = \left(\frac{N_1}{N_1-1}\right)(c - c_{\max}) \\
&< \left(\frac{(N_1-1)c_{\max}^r + c^r}{(N_1-1)c_{\max}^r}\right)(c - c_{\max}) = \left(\frac{1}{1-P_{i1}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})}\right)(c - c_{\max}) \\
\implies &\left(1 - P_{i1}(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})\right)v_f(c, c_{\max}) + (c_{\max} - c) < 0.
\end{aligned}$$

Thus, inequality (14) is satisfied, which means $\pi_i(s_i^*, \boldsymbol{s}^*_{-\boldsymbol{i}}) > \pi(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}})$. □

Theorem 3.8. *Suppose that* $\lambda \geq \frac{\bar{N}}{\bar{N}-1}$ *and* $r > \frac{\ln\left(2+\frac{2}{\bar{N}-2}\right)}{\ln 2}$. *Then* $\boldsymbol{s}^* = \left((\mathcal{N}_1, c_{\max}), (\emptyset, 0)\right)$, *where* $N_1 = \bar{N}$, *is a stationary, conditionally symmetric equilibrium no matter how large* $c_{\max}$ *is. Moreover, the equilibrium is unique up to equivalence class.*

Proof. By Lemma 3.2, there is no SCS equilibrium in which $N_1 + 2N_2 < \bar{N}$. For the $N_1 + 2N_2 \geq \bar{N}$ case, first note that since $r > 0$, $\lambda \geq \frac{\bar{N}}{\bar{N}-1}$ means $r + \lambda > \frac{\bar{N}}{\bar{N}-1}$. Therefore, Lemmas 3.3, 3.4 and 3.5 establish that there is no SCS equilibrium where both $\mathcal{N}_1$ and $\mathcal{N}_2$ are non-empty. Lemmas 3.4 and 3.5 also show that there is no SCS equilibrium in which only $\mathcal{N}_2$ is non-empty, while Lemma 3.3 shows that there is no SCS equilibrium in which only $\mathcal{N}_1$ is non-empty and $c_1 < c_{\max}$. Suppose a SCS strategy profile is such that only $\mathcal{N}_1$ is non-empty, $c_1 = c_{\max}$, and $N_1 > \bar{N}$. The expected payoff of the couples in $\mathcal{N}_1$ in this profile is $\frac{v_s}{N_1} - c_{\max} \leq \frac{v_s}{\bar{N}+1} - c_{\max} < 0$, so this cannot be an equilibrium.

Therefore, the only remaining candidate for an equilibrium is $\boldsymbol{s}^*$, where $\mathcal{N}_1$ is non-empty, $c_1 = c_{\max}$, and $N_1 = \bar{N}$. For any $i \in \mathcal{N}_0$ and $s_i \neq s_i^*$, $\pi_i\left(s_i^*, \boldsymbol{s}^*_{-\boldsymbol{i}}\right) > \pi_i\left(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}}\right)$ by Lemma 3.6. For any $i \in \mathcal{N}_1$ and $s_i = (n, c) \neq s_i^*$, $\pi_i\left(s_i^*, \boldsymbol{s}^*_{-\boldsymbol{i}}\right) \geq \pi_i\left(s_i, \boldsymbol{s}^*_{-\boldsymbol{i}}\right)$ by Lemma 3.7. Moreover, this inequality is strict if $n \geq 1$, which assures that no one will deviate from $\boldsymbol{s}^*$ even under the assumed tie-breaking rule. Thus, $\boldsymbol{s}^*$ is the unique (up to equivalence class) SCS equilibrium. □

Corollary 3.9. *Suppose that* λ *and* r *satisfy the hypothesis of Theorem 3.8. Then the equilibrium fertility rate,* $\bar{N}/N$, *is non-decreasing in* v_s *and non-increasing in* $c_{\max}$.

Proof. Immediate from the definition of $\bar{N}$. □