CHOICE UNDER UNCERTAINTY.

- A LOTTERY (gamble) IS A PROBABILITY DISTRIBUTION OVER A SET OF OUTCOMES.
- E.G. LET $A = \{a_1, a_2, \ldots, a_n\}$ BE A SET OF POSSIBLE MONETARY OUTCOMES.

THEN LOTTERY OVER A HAS THE FORM

$$L = (p_1, p_2, p_3, \ldots, p_n), \quad 0 \le p_k \le 1 \ \forall k$$
$$\& \ \sum_{k=1}^{n} p_k = 1$$

WITH THE INTERPRETATION THAT

p_i = PROB OF OUTCOME i OCCURING.

SOMETIMES, WRITE

$L = (p_1 \circ a_1, p_2 \circ a_2, \ldots, p_n \circ a_n)$ TO MAKE THE MEANING MORE CLEAR.

- FIRST QUESTION: HOW TO ASSIGN PREFERENCE OVER LOTTERIES. WANT $\succsim$ TO BE REPRES. BY UTILITY FUNCTION U.

INITIAL THOUGHT OF USING

$E[L] = \sum_{k=1}^{n} p_k a_k$ TURNS OUT TO BE

NOT GOOD.

TO SEE:

- **ST. PETERSBURG PARADOX.**

SUPPOSE YOU ARE GIVEN AN OPPORTUNITY TO PLAY THE FOLLOWING GAMBLE L:

YOU GET TO REPEATEDLY TOSS A FAIR COIN UNTIL T COMES UP. & YOU ARE PAID ¥ 2^n, $n \equiv$ # OF TOSS.

EG IF

T ⇒ PAID ¥2

HT ⇒ ¥ $2^2 = 4$

HHT ⇒ ¥ $2^3 = 8$, ETC.

HOW MUCH WILL YOU PAY TO PLAY THIS GAMBLE?

MOST ARE WILLING TO PAY $\ll \infty$ DESPITE THE FACT THAT

$$E[L] = \frac{1}{2}(2) + \frac{1}{2^2}(2^2) + \frac{1}{2^3}(2^3) + \cdots$$
$$= \sum_{k=1}^{\infty} 1 = \infty.$$

- **SOLN TO PARADOX**: ASSUME DM HAS A UTILITY FUNCTION $u(\cdot)$ OVER THE SET OF OUTCOMES (MONEY). THEN PLAUSIBLE TO ASSUME

$$U(L) \equiv E[u(L)] \equiv \sum_{k=1}^{n} p_k u(a_k).$$

EXAMPLE: ST. PETERSBURG:

ASSUME DM HAS UTILITY FUNC. $u(x) = x^{1/2}$ OVER MONEY.

THEN $U(L) = E[u(L)]$

$$= \frac{1}{2^1} u(2^1) + \frac{1}{2^2} u(2^2) + \frac{1}{2^3} u(2^3) + \dots$$

$$= \sum_{k=1}^{\infty} \frac{1}{2^k} (2^k)^{1/2} = \sum_{k=1}^{\infty} 2^{-k} 2^{\frac{k}{2}}$$

$$= \sum_{k=1}^{\infty} 2^{-\frac{k}{2}} = \sum_{k=1}^{\infty} \left(\frac{1}{\sqrt{2}}\right)^k$$

$$= \frac{1}{\sqrt{2}} \sum_{k=0}^{\infty} \left(\frac{1}{\sqrt{2}}\right)^k = \frac{1}{\sqrt{2}} \left(\frac{1}{1-\frac{1}{\sqrt{2}}}\right)$$

$$\approx 2.414$$

UTILITY FROM LOTTERY IS 2.414

THIS IS EQUIVALENT TO HAVING $x = (2.414)^2 = £5.828$

$u(x) = x^{1/2} = 2.44$

SO DM WITH $u(x) = x^{1/2}$ IS WILLING TO PAY AT MOST £5.828 TO PLAY THIS LOTTERY.

—//—

SO. GENERAL FRAMEWORK:

- LET $X \subseteq \mathbb{R}$ BE A SET OF POSSIBLE OUTCOMES.
- LET $u: X \to \mathbb{R}$ BE DM'S UTILITY FUNCTION OVER SET OF OUTCOMES (BERNOULLI UTILITY FUNCTION).
- LOTTERY L OVER X CAN BE IDENTIFIED WITH ITS DISTRIBN FUNCTION

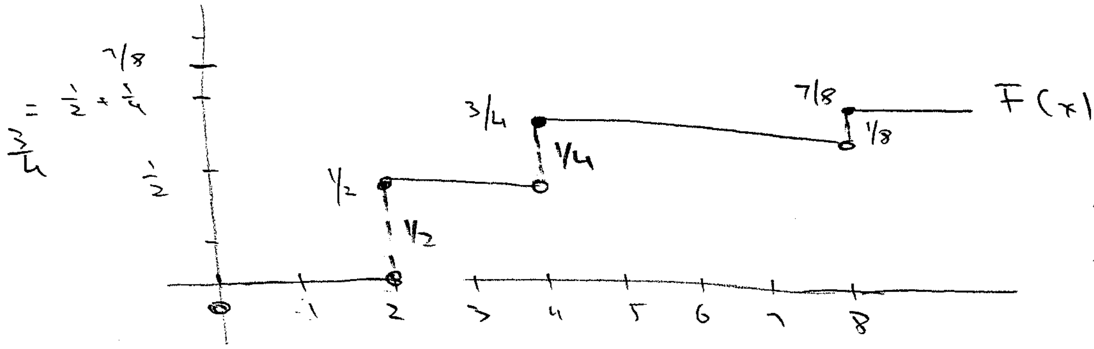
$$dF(x) = \begin{cases} \text{DENSITY } f(x) & \text{IF CONTINUOUS DISTRIBN} \\ \text{MASS FUNC. } p(x) & \text{IF DISCRETE} \end{cases}$$

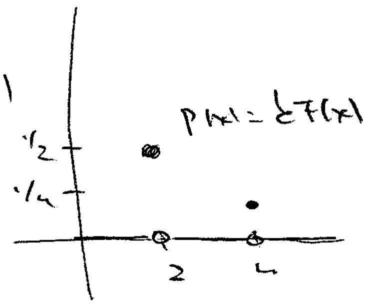
EXAMPLE: L FROM ST. PETERSBURG PARADOX

HAS MASS FUNCTION $p(x) = \frac{1}{2^n}$ IF $x = 2^n$, $n \in \mathbb{Z}_+$

$= 0$ O.W.

& DISTBN FUNCTION

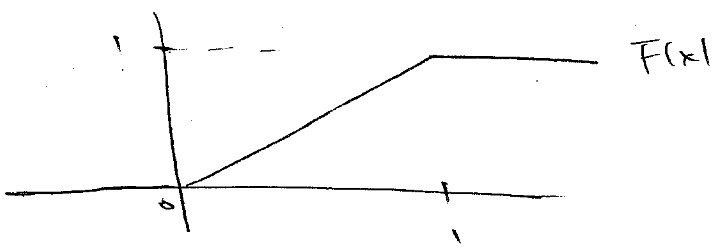


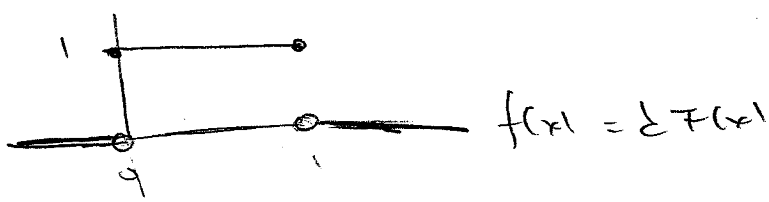


EXAMPLE: L PAYS $x \in [0,1]$ WITH EQUAL PROBABILITY.

THEN $p(x) = 0 \quad \forall x$

But $f(x) = 1$ & $F(x) = \begin{cases} 0 & \text{IF } x \le 0 \\ x & \text{IF } x \in [0,1] \\ 1 & \text{IF } x \geq 1 \end{cases}$





- $\mathcal{L}$ = SET OF ALL POSSIBLE LOTTERIES OVER X.

- ASSUME DM HAS WELL-DEFINED $\succsim$ OVER $\mathcal{L}$ THAT CAN BE REPRESENTED BY UTILITY FUNCTION THAT HAS <u>EXPECTED UTILITY FORM</u> (VON-NEUMANN MORGENSTERN UTILITY FUNC)

I.E. $\forall L, L' \in \mathcal{L}$,

$$L \succsim L' \text{ IFF } U(L) \geq U(L')$$

$$\& \quad U(L) \equiv \int_X u(x)\, dF(x) \equiv E[u(L)]$$

$$\equiv U(F)$$

<u>REMARK</u>: WE DEFER DISCUSSION OF WHEN SUCH $U(\cdot)$ EXISTS TO LATER (6.B IN MWG).

<u>REMARK</u>: IF L IS DEGENERATE, I.E. PAY $\bar{x}$ WITH PROB. 1 THEN

$$U(L) = \int u(x)\, dF(x) = u(\bar{x}).$$

SO, $U(\cdot)$ IS AN <u>EXTENSION</u> OF $u(\cdot)$ INTO SPACE OF LOTTERIES.

<u>REMARK</u>: LINEAR TRANSFORMATION OF $U(\cdot)$,

$\tilde{U}(L) = aU(L) + b$, $a > 0$ (OR EQUIVALENTLY, $\tilde{u}(x) = au(x) + b$) REP. SAME $\succsim$ AS $U(\cdot)$

I.E. $\tilde{U}(L) \geq \tilde{U}(L') \iff U(L) \geq U(L')$

ATTITUDE TOWARDS RISK

CONSIDER $L \begin{cases} \xrightarrow{1/2} 2 \\ \xrightarrow{1/2} 0 \end{cases}$ THEN $E[L] = 1$.

IF DM PREFERS TO HAVE \$1 FOR SURE RATHER THAN THE LOTTERY L, THEN DM SEEMS TO DISLIKE "RISK".

So,

DEFN: • A DM IS ① RISK-AVERSE IF
$E[L] \succsim L$ FOR ALL LOTTERY L.

② STRICTLY RISK-AVERSE IF
$E[L] \succ L$ FOR ALL NON-DEGENERATE LOTTERY L

③ RISK-NEUTRAL IF
$E[L] \sim L$ FOR ALL LOTTERY L

④ RISK-LOVING IF $\precsim$ ...

⑤ S. RISK-LOVING IF $\prec$ ...

ALT. WAYS OF THINKING ABOUT RISK AVERSION

DEFN: GIVEN BERNOULLI UTILITY $u(\cdot)$,
① CERTAINTY EQUIVALENT OF $F(\cdot)$, DENOTED $c(F,u)$, IS DEFINED BY $u(c(F,u)) = \int u(x)\,dF(x) = U(F)$.

I.E. AMT OF MONEY THAT DM WOULD BE INDIFFERENT BETWEEN HAVING IT FOR SURE OR LOTTERY F

(2) FOR ANY FIXED AMT OF MONEY x & $\varepsilon > 0$.

THE **PROBABILITY PREMIUM**, DENOTED $\pi(x, \varepsilon, u)$

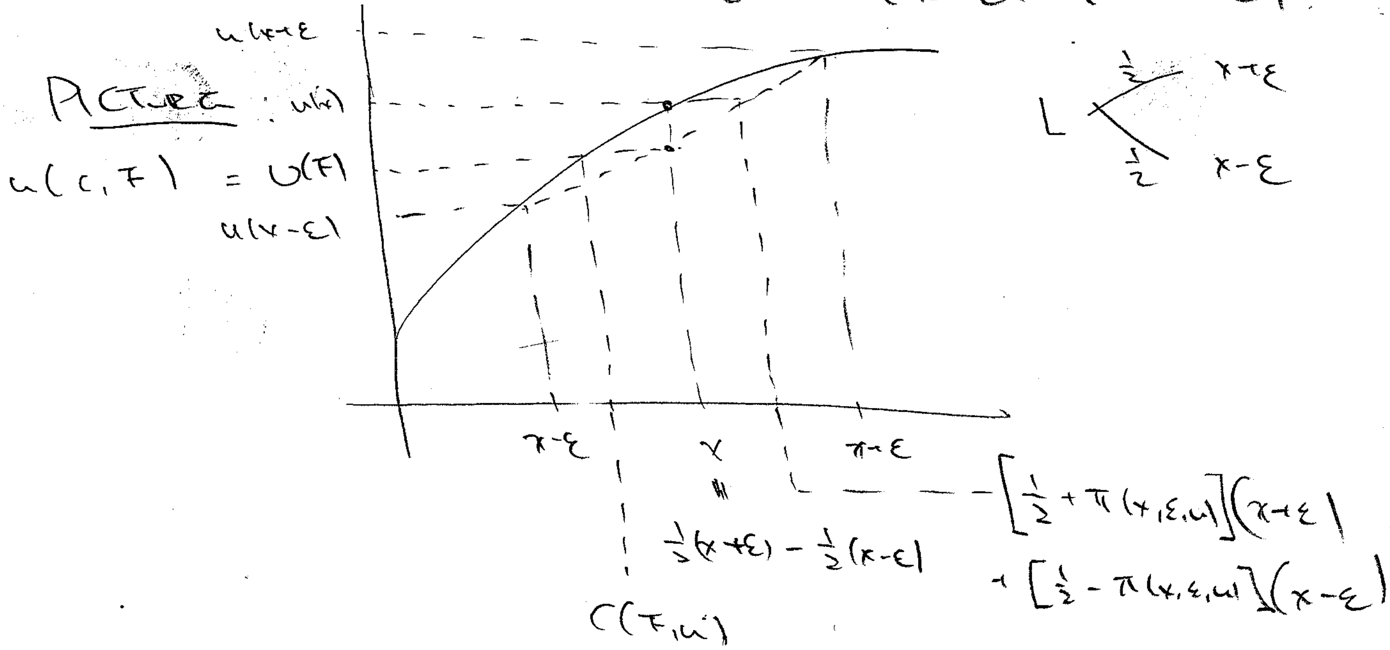
IS DEFINED BY RELATIONSHIP

$$x \sim \left(\tfrac{1}{2} + \pi(x,\varepsilon,u) \circ x+\varepsilon \, , \, \tfrac{1}{2} - \pi(x,\varepsilon,u) \circ x-\varepsilon \right)$$

$$\Leftrightarrow \quad u(x) = \left(\tfrac{1}{2} + \pi(x,\varepsilon,u)\right) u(x+\varepsilon) + \left(\tfrac{1}{2} - \pi(x,\varepsilon,u)\right) u(x-\varepsilon).$$

I.E. THE ADDITIONAL WINNING PROBABILITY ABOVE THE "FAIR" PROBABILITY THAT IS NEEDED BY THE DM TO MAKE CERTAIN OUTCOME x EQUIVALENT TO A LOTTERY BETWEEN $(x+\varepsilon)$ & $(x-\varepsilon)$.

PICTURE:



NOTE: CONCAVE $\Leftrightarrow$ $c(F,u) \leq E[F]$.

$\Leftrightarrow$ $\pi(x,\varepsilon,u) \geq 0$

THIS HOLDS GENERALLY

THM: DM w/ B. UTILITY $u(\cdot)$ IS RISK-AVERSE

$\iff$ ① $u(\cdot)$ IS CONCAVE

$\iff$ ② $c(F, u) \leq E[F] \quad \forall F$

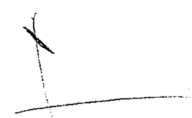
$\iff$ ③ $\pi(x, \varepsilon, u) \geq 0 \quad \forall x \text{ \& } \varepsilon \geq 0$

EXAMPLE: **INSURANCE**: STRICTLY RISK-AVERSE DM w/ INITIAL WEALTH w FACES LOSS OF $D \in (0, w]$ WITH PROB π. INSURANCE IS AVAILABLE. EACH UNIT COST \$$q$ & PAYS \$1 IF LOSS OCCURS & \$0 O.W.

$L(\alpha)$: with prob. π: $w - \alpha q - D + \alpha$; with prob. $1-\pi$: $w - \alpha q$

HOW MUCH INSURANCE TO BUY?

$\alpha \equiv$ AMT OF INSURANCE



DM $\max\limits_{\alpha \geq 0} U(L(\alpha))$

$\iff \max\limits_{\alpha \geq 0} \pi u(w - \alpha q - D + \alpha) + (1-\pi) u(w - \alpha q)$

FOC: $\pi u'(w - \alpha^* q - D + \alpha^*)(1-q) + (1-\pi) u'(w - \alpha^* q)(-\alpha^*) \leq 0$

$[= 0 \text{ IF } \alpha > 0]$

Suppose q is FAIR PRICE SO THAT $q = \pi$.

I.E. $E[\text{PROFIT}] = \pi(\alpha q - \alpha) + (1-\pi)(\alpha q)$

$= \pi \alpha q - \alpha \pi + \alpha q - \pi \alpha q$

$= \alpha(q - \pi) = 0 \quad \text{IF } q = \pi.$

SOC: $\pi u''(w - \alpha^* q - D + \alpha^*)(1-q)^2 + (1-\pi) u''(w - \alpha^* q)(-\alpha^*)^2$

< 0 B/C RISK-AVERSION $\Rightarrow u'' < 0$. SATISFIED

THEN:

FOC: * $\pi u'(w - \alpha^*\pi - D + \alpha^*)(1-\pi) + (1-\pi)u'(w - \alpha^*\pi)(-\pi) \leq 0$

IF $\alpha^* = 0$, THEN

$* = \pi u'(w - D)(1-\pi) + (1-\pi)u'(w)(-\pi) \leq 0$

$u'' < 0 \Rightarrow u'(w-D) > u'(w)$

$\Rightarrow * = \pi(1-\pi)u'(w-D) - \pi(1-\pi)u'(w) > 0$

So, $\not\leq 0$.

$\Rightarrow \alpha^* > 0. \Rightarrow$ FOC $*$ HOLDS W/ EQUALITY.

$\Rightarrow$ FOC: $\pi(1-\pi)u'(w - \alpha^*\pi - D + \alpha^*) = \pi(1-\pi)u'(w - \alpha^*\pi)$

$\Leftrightarrow u'(w - \alpha^*\pi - D + \alpha^*) = u'(w - \alpha^*\pi)$

$\Rightarrow w - \alpha^*\pi - D + \alpha^* = w - \alpha^*\pi$ SINCE $u'' < 0$ OR $u' \downarrow$.

$\Rightarrow \alpha^* = D.$

So, EVERY S. RISK-AVERSE INDIVIDUAL WILL <u>FULLY</u> <u>INSURE</u> IF q IS FAIR.

EVEN THOSE WHO ARE ONLY "TINY-BIT RISK-AVERSE"

<u>REMARK:</u> IF NE, WILL HAVE

$\underline{W - D\pi}$

IF E, WILL HAVE

$W - D\pi - D + D = \underline{W - D\pi}.$

EXAMPLE: DEMAND FOR RISKY ASSET.

ASSET = CLAIM TO ¥ IN THE FUTURE.

TWO ASSETS ① SAFE ASSET

¥1 INVESTED IN SAFE ASSET

RETURNS ¥1 IN FUTURE

② RISKY ASSET

¥1 INVESTED IN RISKY ASSET

RETURNS ¥Z, WHERE Z IS

R.V. w/ DISTBN $F(z)$.

& $\int z \, dF(z) > $ ¥1 !

SUPPOSE S. RISK-AVERSE DM HAS \$W TO INVEST. HOW TO ALLOCATE ¥ BETWEEN THE TWO ASSETS?

LET $\alpha \equiv$ AMT INVESTED IN RISKY ASSET.

THEN RETURN WILL BE $L(\alpha) = \alpha z + (w - \alpha)$

$= w + \alpha(z-1)$ WITH DISTBN $F(\cdot)$.

DM $\max_{0 \le \alpha \le w} U(L(\alpha))$

$\Leftrightarrow \max_{0 \le \alpha \le w} \int u(w + \alpha(z-1)) \, dF(z)$

FOC: (*) $\frac{d}{d\alpha} \int u(w + \alpha(z-1)) \, dF(z)$

$= \int u'(w + \alpha^*(z-1))(z-1) \, dF(z) \begin{cases} \le 0 & \text{IF } \alpha^* = 0 \\ = 0 & \text{IF } \alpha^* \in (0, w) \\ \ge 0 & \text{IF } \alpha^* = w \end{cases}$

SOC: $\frac{d}{d\alpha} * = \int u''(w + \alpha^*(z-1))(z-1)^2 \, dF(z)$

< 0 SINCE S. RISK-AVERSE.

SO FOC IS NEC. & SUFFICIENT

SUPPOSE $\alpha^* = 0$. THEN

$$(*) = \int u'(w)(z-1)\,dF(z)$$

$$= u'(w)\int (z-1)\,dF(z) = u'(w)\left[\left[\int z\,dF(z)\right] - 1\right]$$

$$= > 0$$ SO, FOC NOT SATISFIED @ $\alpha^* = 0$.

$\Rightarrow$ $\alpha^* = 0$ CANNOT BE A SOLUTION

$\Rightarrow$ EVERY RISK-AVERSE INDIVIDUAL, EVEN THOSE THAT ARE VERY RISK-AVERSE, WILL INVEST IN SOME RISKY ASSET

$\Rightarrow$ EVERY R-A DM WILL ACCEPT SOME RISK IF IT IS <u>ACTUARILY</u> <u>FAVORABLE</u>

—//—

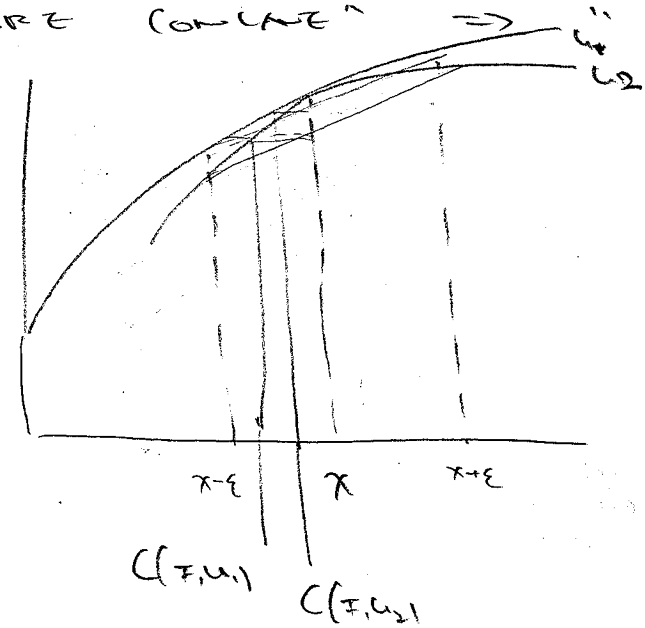
MEASURING RISK-AVERSION

ASSUME B-UTILITY $u(\cdot)$ IS TWICE DIFFERENT.

<u>DEFN</u>: ARROW-PRATT COEFFICIENT OF <u>ABSOLUTE RISK AVERSION</u> AT x IS $r_A(x) \equiv -\dfrac{u''(x)}{u'(x)}$

RECALL RISK-AVERSION $\Leftrightarrow$ CONCAVITY OF $u(\cdot)$

"MORE CONCAVE" $\Rightarrow$ "MORE RISK-AVERSE"



$C(F,u_2) < C(F,u_1)$

HOW TO MEASURE DEGREE OF CONCAVITY?

- REMARK: $u''(\cdot)$ GIVES "CURVATURE" OF $u(\cdot)$.

SINCE $u(x)$ AND $\tilde{u}(x) = a\,u(x) + b$ REPRESENTS THE SAME DM, SHOULD HAVE SAME MEASURE.

$$\tilde{u}''(x) = a\,u''(x) + u'(x)$$

BUT
$$r_A(x, \tilde{u}) = -\frac{\tilde{u}''(x)}{\tilde{u}'(x)} = -\frac{a\,u''(x)}{a\,u'(x)} = -\frac{u''(x)}{u'(x)} = r_A(x, u)$$

- DEFN: WE SAY THAT DM 2 IS MORE RISK-AVERSE THAN DM 1 IF WHENEVER DM 2 PREFERS LOTTERY F TO CERTAIN OUTCOME x THEN SO DOES DM 1 ($F \succsim_2 x \Rightarrow F \succsim_1 x \quad \forall F$)

- THM: DM_2 WITH B. UTILITY $u_2(\cdot)$ IS MORE RISK-AVERSE THAN DM_1 WITH B. UTILITY $u_1(\cdot)$.

$\Leftrightarrow$ ① $r_A(x, u_2) \geq r_A(x, u_1) \quad \forall x$

$\Leftrightarrow$ ② $u_2(\cdot)$ IS A CONCAVE TRANSFORMATION OF $u_1(\cdot)$.
I.E. $u_2(x) = \psi(u_1(x))$ FOR SOME CONCAVE $\psi(\cdot)$

$\Leftrightarrow$ ③ $c(F, u_2) \leq c(F, u_1) \quad \forall F$

$\Leftrightarrow$ ④ $\pi(x, \varepsilon, u_2) \geq \pi(x, \varepsilon, u_1) \quad \forall\, x\ \&\ \varepsilon > 0$

- DEFN: COEFFICIENT OF RELATIVE RISK AVERSION IS

$$r_R(x, u) = -\frac{x\,u''(x)}{u'(x)}.$$

- MEASURES DEGREE OF RISK AVERSION WHEN RISK INVOLVES PROPORTIONAL INC./DEC. FROM CURRENT WEALTH. TO SEE, LET $\tilde{u}(t) = u(tx)$ FOR FIXED x.

THEN
$$r_A(t, \tilde{u}) = -\frac{\tilde{u}''(t)}{\tilde{u}'(t)} = -\frac{x^2 u''(tx)}{x\,u'(tx)}$$
$$= -\frac{x\,u''(tx)}{u'(tx)} = r_R(x, u) \text{ WHEN } t = 1$$

DEFN: DM IS SAID TO EXHIBIT

(1) DECREASING ABSOLUTE / RELATIVE RISK AVERSION

IF $r_A(x,u)$ / $r_R(x,u)$ IS A DEC. FUNC OF x.

DARA / DRRA

(2) CONSTANT ABSOLUTE / RELATIVE RISK AVERSION

IF $r_A(x,u)$ / $r_R(x,u)$ IS A CONSTANT FUNC OF x

CARA / CRRA

(3) INCREASING ABSOLUTE / RELATIVE RISK AVERSION

IF $r_A(x,u)$ / $r_R(x,u)$ IS AN INC. FUNC. OF x.

SEE MWG 6.C FOR MANY EQUIV. WAYS OF THINKING ABOUT DARA / DRRA, ETC.

REMARK: CARA / CRRA UTILITIES ARE USED EXTENSIVELY IN FINANCE THEORY.

EXAMPLE: CONSIDER PORTFOLIO EXAMPLE. ASSUME $\alpha^* \in (0,w)$ & DM IS S. RISK AVERSE & DARA.

SHOW THAT $\frac{d\alpha^*}{dw} > 0$.

FROM FOC: $\frac{d}{dw} \int u'(w+\alpha^*(z-1))(z-1)\,dF(z) = \frac{d}{dw} 0$

$\Rightarrow \int \left[u''(w+\alpha^*(z-1))(z-1) + u''(w+\alpha^*(z-1))(z-1)^2 \frac{d\alpha^*}{dw} \right] dF(z) = 0$

$\Rightarrow \frac{d\alpha^*}{dw} = \frac{-\int u''(w+\alpha^*(z-1))(z-1)\,dF(z)}{\int u''(w+\alpha^*(z-1))(z-1)^2\,dF(z)}$ DENO (−) BY SOC.

NUM $= \int -\frac{u''(w+\alpha^*(z-1))(z-1)}{u'(w+\alpha^*(z-1))} u'(w+\alpha^*(z-1))\,dF(z)$

$= \int R_A(w+\alpha^*(z-1))(z-1)\,u'(w+\alpha^*(z-1))\,dF(z)$

$z>1 \Rightarrow R_A(w+\alpha^*(z-1))(z-1) < R_A(w)(z-1)$

$z<1 \Rightarrow R_A(w+\alpha^*(z-1))(z-1) < R_A(w)(z-1)$

$\Rightarrow\ < \int R_A(w)(z-1)\,u'(w+\alpha^*(z-1))\,dF(z) = R_A(w) \int u'(w+\alpha^*(z-1))(z-1)\,dF(z)$

$= 0$. So, $\frac{d\alpha^*}{dw} = \frac{(-)}{(-)} > 0$. //

STOCHASTIC DOMINANCE:

HOW TO DETERMINE WHEN ONE LOTTERY IS BETTER THAN ANOTHER ⟨FOR A WIDE CLASS OF DM⟩

RESTRICT TO DISTRN WITH $F(0) = 0$ & $F(\bar{x}) = 1$ FOR SOME $\bar{x}$

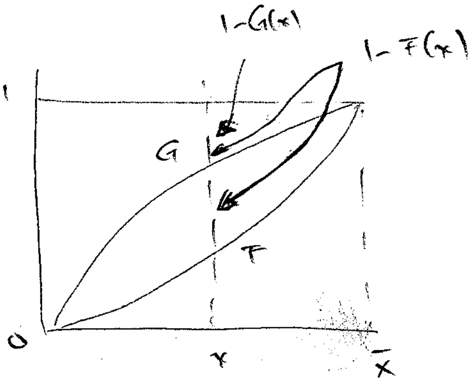
DEFN: $F(\cdot)$ FIRST ORDER STOCHASTICALLY DOMINATE (FOSD) $G(\cdot)$ IF $F(x) \le G(x) \quad \forall x$.

I.E. PROB OF GETTING HIGHER THAN x UNDER $F(\cdot)$ $= 1 - F(x)$

$\geq 1 - G(x)$ = PROB OF GETTING HIGHER THAN x UNDER G.

FOR ALL x.

PICTURE:

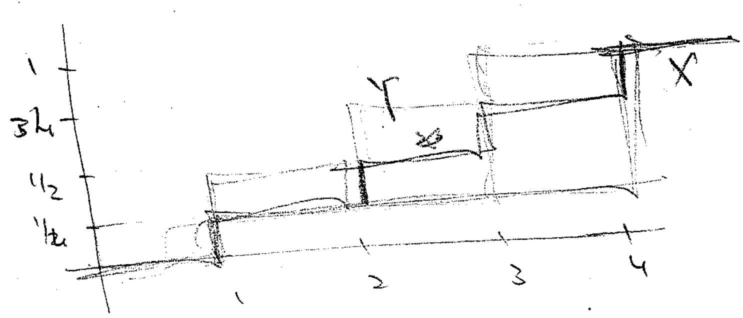


THM: $F(\cdot)$ FOSD $G(\cdot)$ $\Leftrightarrow$ $\forall$ NON-DECREASING $u(\cdot)$, $\int u(x)\,dF(x) \geq \int u(x)\,dG(x)$

I.E. EVERY EU MAXIMIZER WHO PREFERS MORE TO LESS W. PREFERS $F(\cdot)$ TO $G(\cdot)$

X

Y



$X = (\tfrac{1}{4}\circ 1, \tfrac{3}{4}\circ 4)$

$Y = (\tfrac{1}{4}\circ 1, \tfrac{1}{4}\circ 2, \tfrac{1}{4}\circ 3, \tfrac{1}{4}\circ 4)$

DEFN: SAY THAT $G(\cdot)$ IS A MEAN PRESERVING SPREAD OF $F(\cdot)$ IF $G(\cdot)$ CAN BE OBTAINED FROM $F(\cdot)$ BY FOLLOWING PROCEDURE

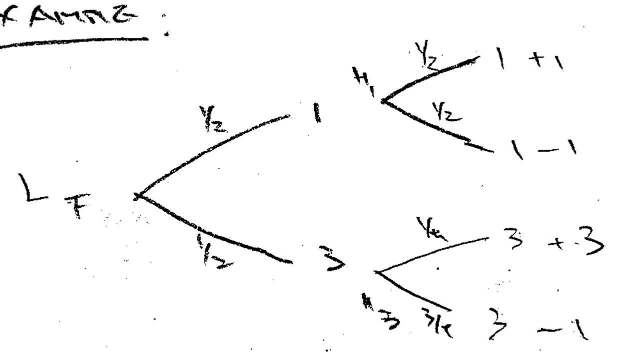
AFTER DRAWING OUTCOME x ACCORDING TO $F(\cdot)$, DRAW z ACCORDING TO DISTBN $H_x(\cdot)$ WHICH HAS MEAN ZERO.

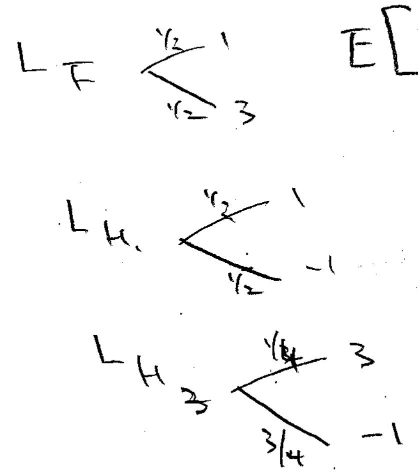
IE. $$G(x+z) = \int_0^x \left[\int_0^z x'+z' \, dH_{x'}(z')\right] dF(x')$$

WHERE $\int z \, dH_x(z) = 0 \quad \forall x$.

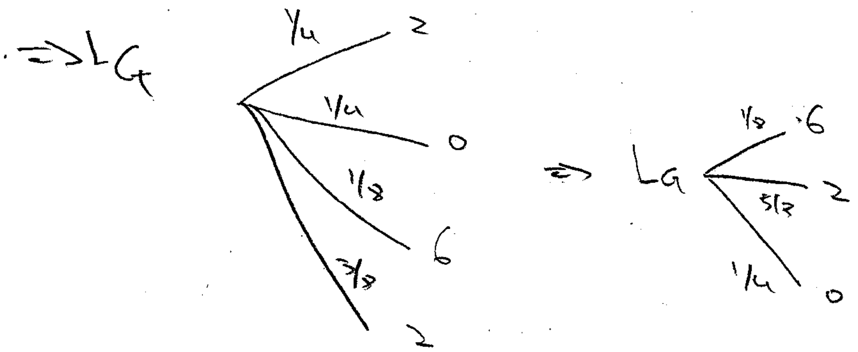
OR, $$G(x) = \int_0^x \int_0^{x-x'} x'+z' \, dH_{x'}(z') \, dF(x')$$

EXAMPLE:

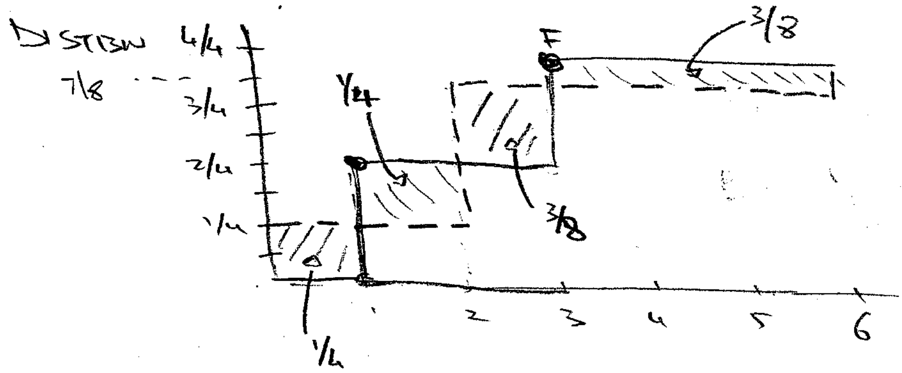


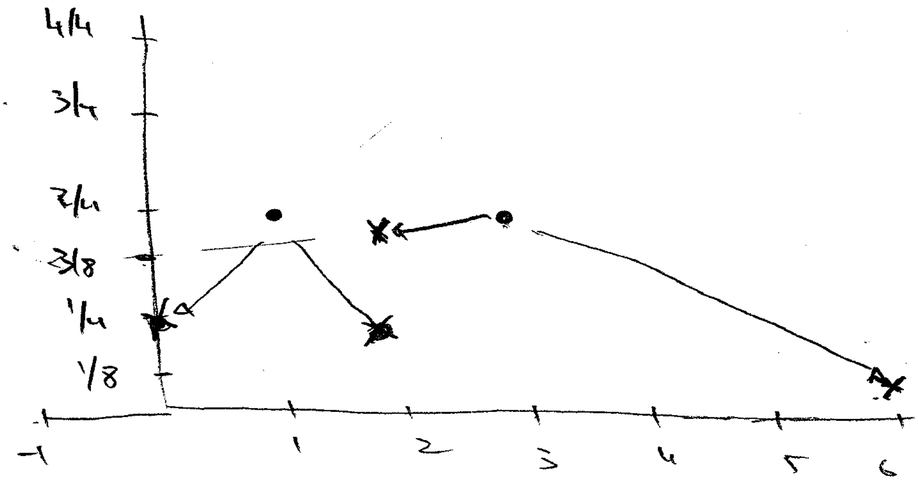


$E[L_F] = 2$



$E[L_G] = \frac{16}{8} = 2$





G TAKES LOTTERY F & FURTHER GARBLES IT W/O IMPROVING ITS MEAN.

DEFN: F(·) SECOND ORDER STOCHASTICALLY DOMINATE (SOSD) G(·) IF G(·) IS A MEAN PRESERVING SPREAD OF F(·)

THM: F(·) SOSD G(·) IFF EVERY RISK-AVERSE DM WHO W. PREFERS MORE TO LESS W. PREFERS F OVER G.

I.E. $\int u(x)\,dF(x) \geq \int u(x)\,dG(x)$ $\forall$ CONCAVE NON-DEC $u(\cdot)$.

STATE DEPENDENT UTILITY

SO FAR, DM'S UTILITY DEPENDED ON MONETARY OUTCOME & NOT THE EVENT THAT LED TO THAT OUTCOME.

E.G. IN THE INSURANCE EXAMPLE IF THE LOSS OCCURRED AS A RESULT OF EARTHQUAKE / DIVORCE ETC., MAY BE UTILITY OVER MONEY HAS CHANGED B/C OF DEPRESSION ETC.

SO, OFTEN, WANT TO ALLOW UTILITY TO DEPEND ON THE CAUSE OF THE OUTCOME.

THIS CAN BE EASILY INCORPORATE BY EXPLICITLY INTRODUCING "STATES".

LET $S = \{s_1, s_2, \ldots, s_n\}$ BE THE POSSIBLE STATES OF THE WORLD

E.G. $S = \{$ EARTH QUAKE, NO EARTH QUAKE $\}$

AND LOTTERY $L : S \to \mathbb{R}_+$ WITH DISTN FUNCTION $F(\cdot)$.

E.G. $L(E) = W - D$ & $L(NE) = W$

$PROB(E) = \pi$ $\quad$ $PROB(NE) = 1 - \pi$.

- WE CAN REPRESENT LOTTERY AS A POINT ON THE $\mathbb{R}^n_+$ SO THAT $\mathcal{L} = \mathbb{R}^n_+$ & $\succsim$ OVER $\mathcal{L}$ IS A PREF. OVER $\mathbb{R}^n_+$ JUST AS WE HAD DONE FOR COMMODITY SPACE IN CONSUMER THEORY.

INSURANCE EXAMPLE: LET E = STATE 1 NE = STATE 2

& LOTTERY $x = (x_1, x_2)$ REPRESENT HAVING ¥ x_1 IF STATE 1 OCCURS & HAVING ¥ x_2 IF STATE 2 OCCURS.

RECALL $U(x) = \pi u(x_1) + (1-\pi) u(x_2)$

$\Rightarrow$ SLOPE OF INDIFF CURVE IS FOUND BY DIFF $U(x) = 0$

$$\pi \frac{\partial u(x_1)}{\partial x_1} \frac{dx_1}{dx_1} + (1-\pi) \frac{\partial u(x_2)}{\partial x_2} \frac{dx_2}{dx} = 0$$

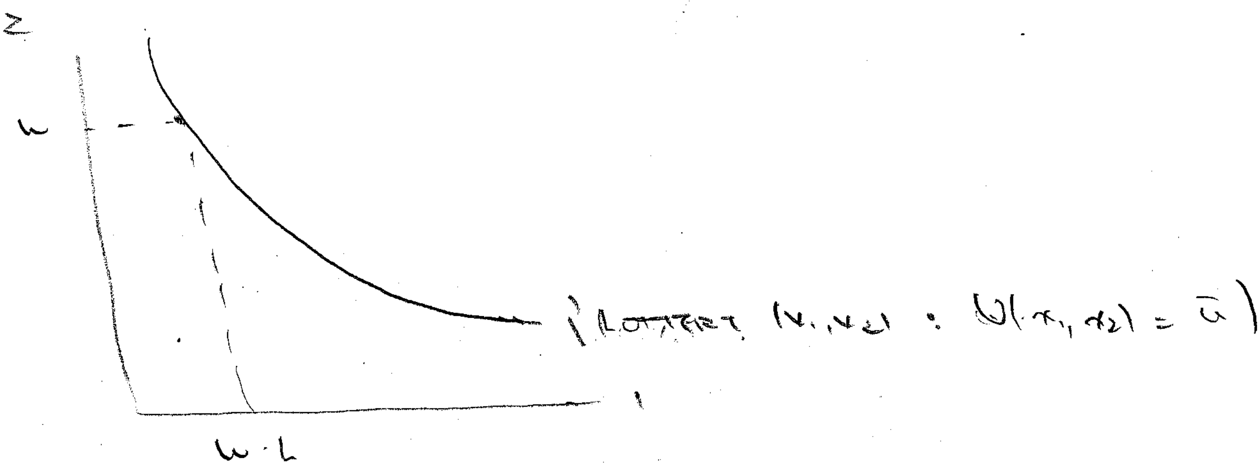
$$\Rightarrow \frac{dx_2}{dx_1} = -\frac{\pi}{(1-\pi)} \frac{\partial u/\partial x_1}{\partial u/\partial x_2} < 0$$

$$\& \quad \frac{d}{dx_1}\left[\frac{dx_2}{dx_1}\right] = -\frac{\pi}{(1-\pi)}\left[\frac{\partial^2 u/\partial x_1^2 \, \frac{\partial u}{\partial x_2} - \frac{\partial u}{\partial x_1} \frac{\partial^2 u}{\partial x_2^2} \frac{\partial x_2}{\partial x_1}}{\left[\partial u/\partial x_2\right]^2}\right]$$

$$= (-)\left[\frac{(-)(+) - (+)(-)(-)}{(+)}\right] = (+).$$

USING S. RISK AVERSE $\Rightarrow u'' < 0$.

SO INDIFF CURVE IS DOWNWARD SLOPING & CONVEX



DM U-MAX PROBLEM <PRICE $q = \pi$>

$$\max_{\alpha} \ \pi u(w - \alpha\pi - L + \alpha) + (1-\pi) u(w - \alpha\pi)$$

$$\Leftrightarrow \max_{x_1, x_2} \ \pi u(x_1) + (1-\pi) u(x_2)$$

S.T. $x_1 = w - \alpha\pi - D + \alpha$

& $x_2 = w - \alpha\pi$

B.C. $\Rightarrow$ $x_1 = (w - D) + \alpha(1-\pi)$

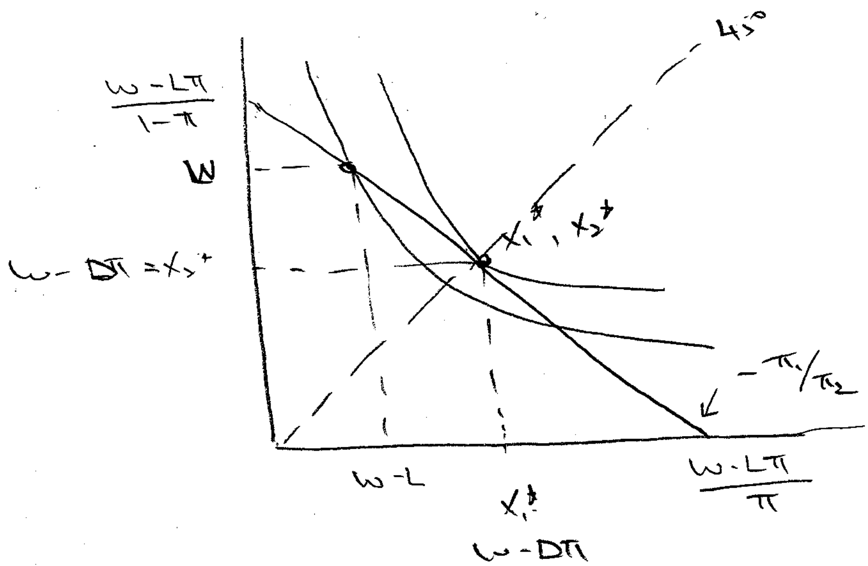
$$\Rightarrow \alpha = \frac{x_1 - (w-D)}{1-\pi}$$

$$\Rightarrow x_2 = w - \left[\frac{x_1}{1-\pi} - \frac{(w-D)}{1-\pi}\right]\pi$$

$$(1-\pi)x_2 = (1-\pi)w - \pi x_1 + (w-D)\pi$$

$\Rightarrow$ $\pi x_1 + (1-\pi)x_2 = w - D\pi$ <EXPECTED $w = w - D\pi$>

So, $\max_{x_1, x_2} \ \underbrace{\pi u(x_1) + (1-\pi) u(x_2)}_{U(x_1, x_2)}$ S.T. $\pi x_1 + (1-\pi) x_2 = w - D\pi$



AT $x_1 = w - D$

$x_2 = w$

$$MRS(w-D, w) = \frac{\pi u'(w-D)}{(1-\pi) u'(w)} > \frac{\pi}{1-\pi}$$

AT x_1^*, x_2^*

FOC

$$\frac{\pi u'(x_1^*)}{1-\pi \ u'(x_2^*)} = \frac{\pi}{1-\pi}$$

$$\frac{u'(x_1^*)}{u'(x_2^*)} = 1$$

$\Rightarrow x_1^* = x_2^*$

$$\pi x_1^* + (1-\pi) x_1^* = w - D\pi$$

$$x_1^* = w - D\pi$$

$\alpha = D$

INSURANCE ALLOWS ONE TO TRANSFER WEALTH ACROSS STATE. (ASSET) S.R-AVERSE AGENT IS MADE S. BETTER OFF!

NOW SUPPOSE UTILITY OVER MONEY IS
ALLOWED TO DEPEND ON STATE

IN STATE 1 HAVE $u_1(\cdot)$ $\quad u_s' > 0$ $\forall_s$

& IN STATE 2 HAVE $u_2(\cdot)$ $\quad u_s'' < 0$

ASSUME $u_1'(x) > u_2'(x) \quad \forall x$

THEN DM

$$\max_{x_1, x_2} \pi u_1(x_1) + (1-\pi) u_2(x_2) \quad \text{s.t.} \quad \pi_1 x_1 + \pi_2 x_2 = W - D\pi$$

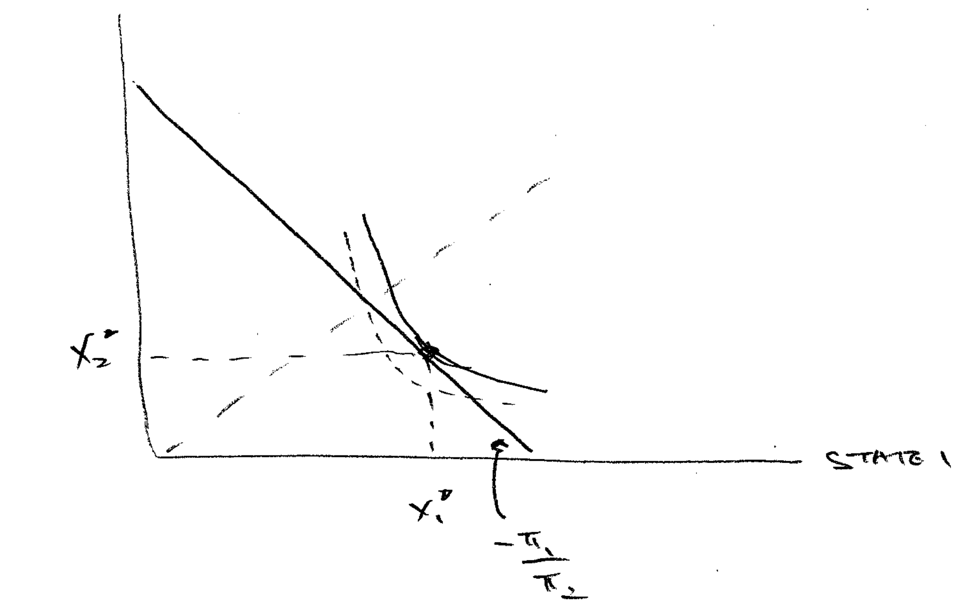
SOLN: $\text{MRS} = \dfrac{\pi}{1-\pi} \dfrac{u_1'(x_1^*)}{u_2'(x_2^*)} = \dfrac{\pi}{1-\pi}$

$\Rightarrow u_1'(x_1^*) = u_2'(x_2^*)$ $\qquad$ SINCE $u_1'(x) > u_2'(x) \ \forall x$

IF $x_1^* = x_2^*$ $\qquad$ & $u_1'' \downarrow$ & $u_2'' \downarrow$

$\therefore$ MRS > PRICE RATIO $\qquad$ MUST HAVE $x_1^* > x_2^*$

$\Rightarrow$



DM WILL OVER INSURE $\alpha^* > D$

MAKES SENSE SINCE $ IS MORE VALUABLE
IN STATE 1.