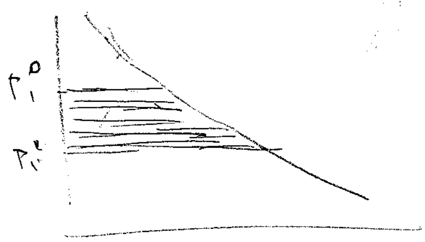
WELFARE ANALYSIS (ARISING FROM PRICE CHANGE)

- INTERMEDIATE MICRO: USE AREA UNDER MARSH. DEMAND AS A MEASURE OF WELFARE
  CONSUMER SURPLUS. LET $p_{-\ell} = (p_1, p_2, \dots, p_{\ell-1}, p_{\ell+1}, \dots p_n)$
  $\Rightarrow p = (p_\ell, p_{-\ell})$



CONSUMER SURPLUS OF p_1^0 & p_1^1 IS

$$+\int_{p_1^1}^{p_1^0} x_1(p_1, p_{-1}^0, w)\, dp_1$$

- BUT WHY DOES THIS GIVE CORRECT MEASURE OF WELFARE CHANGE?
  SOME VAGUE ANSWER ABOUT WILLINGNESS TO PAY VS. WHAT THEY ACTUALLY PAY?
  SHOULD WELFARE MEASURE BE BASED ON THE CONCEPT OF UTILITY?

- TRY TO BUILD A "BETTER MEASURE".
  WANT TO COMPARE (p^0, w) VS. (p^1, w)
  HOW TO MEASURE UTILITY VALUE OF PRICE-WEALTH COMBINATION?
  WHY NOT USE INDIRECT UTILITY $v(p, w)$?

NOW, SINCE AFTER ORDINAL RANKS ONLY, LET'S USE STRICTLY INCREASING TRANSFORMATION

$e(p^0, \cdot)$,

I.E. USE $e(p^0, v(p,w)) : (p, w) \longmapsto$ ¥
< MONEY METRIC IND. UTILITY>

USEFUL: SINCE GIVES UTILITY MEASURED IN ¥
AMT OF MONEY IT WOULD TAKE AT CURRENT PRICE LEVEL p^0 TO ACHIEVE UTILITY LEVEL (p, w).

E.G.: CONSIDER PRICE CHANGE FROM p^0 TO p'

AT p^0, CONSUMER ACHIEVED UTILITY LEVEL $v(p^0, w)$

& IT WOULD TAKE HER $e(p^0, v(p^0, w))$ AMT OF WEALTH TO ACHIEVE THAT UTILITY.

AT p', CONSUMER CAN ACHIEVE UTILITY LEVEL $v(p', w)$ & AT CURRENT PRICE LEVEL p^0, IT WILL COST HER $e(p^0, v(p', w))$ TO ACHIEVE THAT.

SO, CHANGE FROM p^0, p' IS WORTH

$$e(p^0, v(p', w)) - e(p^0, v(p^0, w)) \text{ TO HER.}$$

IF > 0 BETTER OFF UNDER p'

& IF < 0 BETTER OFF UNDER p^0.

THIS PARTICULAR METHOD OF MEASURING WELFARE CHANGE IS CALLED EQUIVALENT VARIATION (EV)

$$EV(p^0, p', w) = e(p^0, v(p', w)) - e(p^0, v(p^0, w))$$

$$= e(p^0, v(p', w)) - w.$$

$\approx$ ^max. AMT OF MONEY THE CONSUMER IS WILLING TO ACCEPT / ~~INSTEAD~~ ~~OF~~ p' <IF POSITIVE> OR ^MAX PAY TO AVOID p' <IF NEGATIVE>.

$\approx$ AMT OF \$ THAT IS EQUIVALENT TO Δ WELFARE FROM PROPOSED PRICES p'.

$$\left\langle v(p^0, w + EV) = v(p', w) \right\rangle$$

$= v(p^0, e(p^0, v(p', w)))$

$v(p^0, e(p, w)) = w$

- When we transformed the $v(p,w)$ to get a utility measured in ¥, $\langle e(p^0, \cdot)\rangle$ could have used any price rather than p^0.
- Another obvious price vector to use is p^1.

$\Rightarrow$ Compensating Variation (CV)

$$CV(p^0, p^1, w) = e(p^1, v(p^1,w)) - e(p^1, v(p^0,w))$$

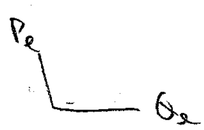
$$= w - e(p^1, v(p^0,w))$$

= max amt of money the consumer is willing to pay (if positive) or min needs to be paid (if negative) to accept p^1.

$\approx$ − amt of compensation consumer demands if p^1 is to be accepted.

$$v(p^1, w - CV) = v(p^1, e(p^1, v(p^0,w))) = v(p^0,w)$$

- Picture: Suppose $p^0_\ell > p^1_\ell$, $p^0_k = p^1_k = \bar{p}_k \ \forall k \neq \ell$ & ℓ is strictly normal. So that $\frac{\partial h_\ell}{\partial p_\ell} > \frac{\partial x_\ell}{\partial p_\ell}$

$\Rightarrow$ On  , graph steeper. $\frac{\partial h_\ell}{\partial p_\ell} = \frac{\partial x_\ell}{\partial p_\ell} + \frac{\partial x_\ell}{\partial w} x_\ell$

$\frac{\partial x_\ell}{\partial w} > 0 \Rightarrow \frac{\partial h_\ell}{\partial p_\ell} > \frac{\partial x_\ell}{\partial p_\ell}$, less negative, steeper on P_ℓ–Q_ℓ curve

Let $u^1 = v(p^1, w)$ & $u^0 = v(p^0, w)$

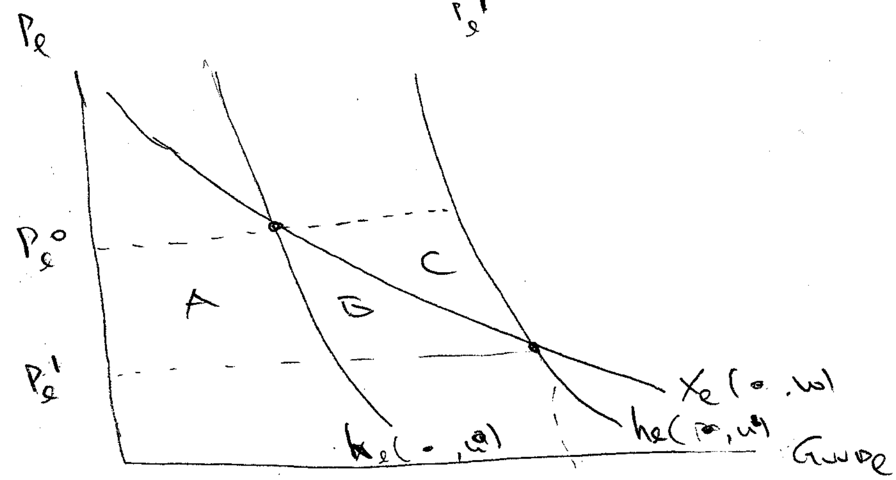
$u' = v(p', w)$

$u^0 = v(p^0, w)$

THEN
$$
\begin{aligned}
EV &= e(p^0, u') - w \\
&= e(p^0, u') - e(p', u') \\
&= \int_{p_\ell'}^{p_\ell^0} \frac{\partial e(p, u')}{\partial p_\ell} \, dp_\ell \\
&= \int_{p_\ell'}^{p_\ell^0} h_\ell(p, u') \, dp_\ell .
\end{aligned}
$$

& $$
\begin{aligned}
CV &= w - e(p', u^0) \\
&= e(p^0, u^0) - e(p', u^0) \\
&= \int_{p_\ell'}^{p_\ell^0} h_\ell(p, u^0) \, dp_\ell .
\end{aligned}
$$

So,



CV = A

CS = AREA VARIATION A + B

EV = A + B + C .

- So, EV > CS > CV (SAME IF $p_\ell^0 < p_\ell'$)
- IF ℓ IS INFERIOR HAVE

EV < CS < CV.

- IF NO <u>WEALTH</u> <u>EFFECT</u> (VERY SPECIAL CASE)

EV = CS = CV

WE HAVE SEEN EV, CS, & CV ALL GIVE $^{\text{DIFF}}$ VALUES IN GENERAL.

SO, MAGNITUDE IS DIFF. HOW ABOUT RANKING?

FACT: WHEN COMPARING p^0 & p^1 WHERE ONLY THE PRICE OF SINGLE GOOD CHANGES, EV, CS, & CV ALL GIVE CORRECT RANKING.

FACT: SUPPOSE WEALTH EFFECT IS NON-ZERO. THEN WHEN COMPARING p^0 & p^1 WHERE PRICES OF MULTIPLE GOODS CHANGE OR WHEN COMPARING MULTIPLE PRICE VECTORS p^0, p^1, p^2, ETC., EV GIVES CORRECT RANKING, BUT CV & CS MAY NOT.

REMARK: CS IS AN APPROXIMATE MEASURE.

II AGGREGATION.

① AGGREGATE DEMAND (AD) & AGG. WEALTH

SUPPOSE HAVE I CONSUMERS, EACH WITH RATIONAL $\succsim_i$ & CORRESP. DEMAND FUNC $X_i(p, w_i)$

THEN $AD = \sum_{i=1}^{I} x_i(p, w_i)$

$= x(p, w_1, w_2, \ldots, w_I)$ IS A FUNC OF p & INDIVIDUAL WEALTH LEVELS $w_1, \ldots, w_I$.

Q: WHAT ASSUMES ON $\succsim_i$'s ALLOW US TO WRITE AD AS A FUNCTION OF p & AGG WEALTH $w = \sum w_i$ ONLY?

I.E. WHEN IS AD DEPEND ONLY ON p & AGG WEALTH RATHER THAN COMPOSITION OF WEALTH? WHEN IS $X(p, w)$ MEANINGFUL?

E.G.: $w_1 = 1$ $\quad x_{\ell 1}(p, w_1) = 5$ $\quad w_1' = 2$, $x_{\ell 1}(p, w_1') = 7$

$w_2 = 3$ $\quad x_{\ell 2}(p, w_1) = 4$ $\quad w_2' = 2$, $x_{\ell 2}(p, w_2') = 3$

THEN CAN'T WRITE AGG DEMAND AS $x(p, 4)$

SINCE $\underbrace{x_{\ell 1}(p, 1) + x_{\ell 2}(p, 3)}_{w=4} = 9 \neq 10 = \underbrace{x_{\ell 1}(p, 2) + x_{\ell 2}(p, 2)}_{w=4}$