II. AGGREGATION.

① AGGREGATE DEMAND (AD) & AGG. WEALTH

SUPPOSE HAVE I CONSUMERS, EACH WITH RATIONAL $\succsim_i$ & CORRESP. DEMAND FUNC $x_i(p, w_i)$.

THEN $AD = \sum_{i=1}^{I} x_i(p, w_i)$

$= x(p, w_1, w_2, \ldots, w_I)$ IS A FUNC OF p & INDIVIDUAL WEALTH LEVELS $w_1, \ldots, w_I$.

Q: WHAT ASSUMES ON $\succsim_i$'s ALLOW US TO WRITE AD AS A FUNCTION OF p & AGG WEALTH $w = \sum w_i$ ONLY?

I.E. WHEN IS AD DEPEND ONLY ON p & AGG. WEALTH RATHER THAN COMPOSITION OF WEALTH? WHEN IS $x(p, w)$ MEANINGFUL?

E.G.: $w_1 = 1$ $\quad x_{\ell 1}(p, w_1) = 5$ $\quad w_1' = 2$, $x_{\ell 1}(p, w_1') = 7$

$w_2 = 3$ $\quad x_{\ell 2}(p, w_2) = 4$ $\quad w_2' = 2$, $x_{\ell 2}(p, w_2') = 3$

THEN CAN'T WRITE AGG DEMAND AS $x(p, 4)$

SINCE $\underbrace{x_{\ell 1}(p,1) + x_{\ell 2}(p,3)}_{w=4} = 9 \neq 10 = \underbrace{x_{\ell 1}(p,2) + x_{\ell 2}(p,2)}_{w=4}$

- SUPPOSE INDIVIDUAL WEALTH IS GENERATED FROM AGG WEALTH THROUGH SOME DISTRIBUTION RULE.

$w_i = w_i(p, w)$ WHERE $w_i : (p, w) \rightarrow w_i$

THEN, OFCOURSE WE HAVE

$$AD = \sum_{i=1}^{I} X_i(p, w_i) = \sum_{i=1}^{I} x_i(p, w_i(p, w))$$

$$= \sum_{i=1}^{I} x_i(p, w) = X(p, w).$$

- BUT, MORE GENERALLY, NEED:

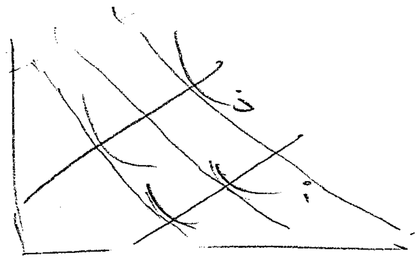
FROM ANY INITIAL WEALTH DISTBN $(w_1, w_2, \ldots w_I)$

AND DISPLACEMENTS $dw_1, dw_2, \ldots, dw_I$ SUMMING UP TO ZERO,

HAVE $\sum_i \frac{\partial x_{\ell i}(p, w_i)}{\partial w_i} dw_i = 0 \quad \forall \ell.$

$\Rightarrow$ TRUE IFF $\frac{\partial x_{\ell i}(p, w_i)}{\partial w_i} = \frac{\partial x_{\ell j}(p, w_j)}{\partial w_j} \quad \forall i \,\&\, j$

THAT IS WEALTH EXPANSION PATHS ARE STRAIGHT & PARALLEL



- $\underline{\text{THM}}$. SET OF CONSUMERS WILL HAVE STRAIGHT PARALLEL WEALTH EXPANSION PATHS AT ANY PRICE p **IFF** THE CONSUMERS ALL HAVE INDIRECT UTILITY OF THE GORMAN FORM

$$V_i(p, w_i) = a_i(p) + \underbrace{b(p)}_{\text{SAME FOR ALL } i} w_i$$

VERY STRONG ASSUMPTION

- ALSO $w_i = w(p, w)$. E.g. $w_i = \frac{\sum_{j=1}^{N} w_j}{N}$

(2) WHAT PROP. OF INDIVIDUAL DEMAND FUNCTION CARRY OVER TO AD?

ASSUME WEALTH DISTBN RULE

- HDO — YES
- CONTINUITY — YES
- W.L. — YES
- N.S.D SLUTSKY MATRIX (COMP. LAW OF DEMAND) — NO.

ULD ⇒ $D_p x$ n.s.d.

ULD ⇐ $D_p x$ n.d.

<u>PROP</u>: IF EVERY CONSUMER'S MARS. DEMAND SATISFIES UNCOMPENSATED (UNRESTRICTED) LAW OF DEMAND THEN AD SATISFIES ULD.

<u>REMARK</u>: $x_i(p, w_i)$ SATISFIES <u>ULD</u> IF ~~ULD~~ n.s.d. ~~$D_p x$~~

$$(p' - p) \cdot [x_i(p', w_i) - x_i(p, w_i)] \leq 0 \quad \forall\, p, p' \; \& \; w.$$

I.E. $(p_\ell' - p) * (x_{\ell i}(p', w_i) - x_{\ell i}(p, w_i)) \leq 0$

IF ONLY p_ℓ Δ TO p_ℓ'

(3) WHEN IS THERE A REPRESENTATIVE CONSUMER FOR THE ECONOMY?

I.E. HAVE AD $x(p, w)$. WANT TO SAY THERE EXISTS A CONSUMER $\succsim$ WITH DEMAND $x(p, w)$ SO THAT WELFARE ANALYS PERFORMED USING $\succsim$ WILL BE VALID FOR AGG WELFARE

<u>A</u>: WHEN WEALTH DISTBN RULE $w_i(p, w)$ SOLVES

<u>SWMP</u>: $\max_{w_1, \ldots, w_I} W(v_1(p, w_1), v_2(p, w_2), \ldots, v_I(p, w_I))$ s.t. $\sum_{i=1}^{I} w_i = W$

STRONG ASSUMPTION — " —

$(p_\ell' - p_\ell) \cdot (x_{\ell i}(p, w_i) - (p_\ell' - p_\ell) \cdot x_{\ell i}(p, w_i)$

③ AD & EXISTENCE OF A REPRESENTATIVE CONSUMER

Q: CONSIDER AN ECONOMY WITH N INDIVIDUALS.

WHEN CAN WE SAY (p, w) IS BETTER / WORSE FOR THE ECONOMY THAN (p', w')?

AGG. WEALTH

IF THERE IS A SINGLE AGENT, THEN USE $V(p, w)$, TRANSFORM IT WITH $e(\cdot, v(p,w))$ TO DERIVE CV, EV, ETC.

WHAT ABOUT IF THERE ARE N INDIVIDUALS?

FIRST DEFINE WHAT MEAN BY BETTER / WORSE FOR THE ECONOMY:

NEED TO DEFINE $V(p, w)$ FIRST

LET: $W : \mathbb{R}^n \to \mathbb{R}$ BE THE SOCIAL WELFARE FUNCTION THAT ASSIGNS "SOCIAL UTILITY" TO THE PROFILE $(u_1, u_2, \ldots, u_n)$

EXAMPLE $W = u_1 + u_2 \cdots + u_n$

$W' = \min\{u_1, u_2, \ldots, u_n\}$

THEN $W(8, 1, 1) = 10 > 9 = W(3, 3, 3)$

BUT $W'(8, 1, 1) = 1 < 3 = W'(3, 3, 3)$

RECALL $(w_1(p, w), w_2(p, w), \ldots, w_n(p, w))$ IS AN WEALTH DIST'BN RULE.

SUPPOSE INSTEAD OF BEING ARBITRARY,
W. DISTBN RULE SOLVES, FOR EACH (p, w)

$$\max_{w_1, \dots, w_N} W(v_1(p, w_1), \dots, v_N(p, w_N))$$

$$\text{s.t. } \sum w_i \leq w.$$

LET $w_1^*(p, w), \dots, w_N^*(p, w)$ BE THE SOLUTION. <OPTIMAL WEALTH DISTBN RULE>

LET $v(p, w) = W(v_1(p, w_1^*), \dots, v_N(p, w_N^*))$
BE THE VALUE FUNCTION.

THEN $v(p, w)$ IS THE INDIRECT UTILITY OF A POSITIVE CONS. WITH DEMAND FUNCTION $x(p, w) = \sum x_i(p, w_i^*(p, w))$.